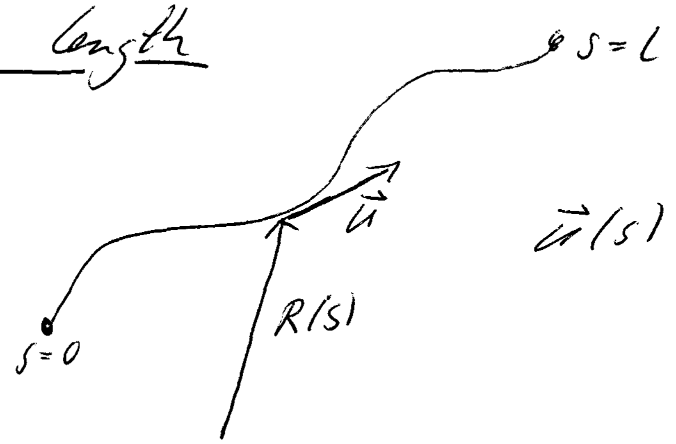
# 3b. Stiffness of Cytoskeletal Filaments

Lit.: • Doi and Edwards, Theory of Polymer Dynamics, Chapt. 8.8
• Käs et al., Biophys. J, Vol 70, 609-625, Feb 1996
• Isambert et al, J. Biol. Chem, Vol 270, 11437-44, May 1995
• LeGoff et al, PRL, Vol 88, 2002

## Fundamental Problem:

• synthetic ~~[illegible]~~ polymers are very flexible
⇔ they only form mechanically stable networks at high volume fraction
• nature needs networks with large mesh sizes
⇒ biopolymers are much stiffer (DNA × 10, Cytoskeleton > × 100)

## Persistence length




$\vec{u}(s) = \frac{\partial \vec{R}}{\partial s}$ tangent vector

bending energy: $\boxed{U_{bend} = \frac{1}{2} k_c \int_0^L \left(\frac{\partial \vec{u}}{\partial s}\right)^2 ds}$

distribution of $\vec{u}$:

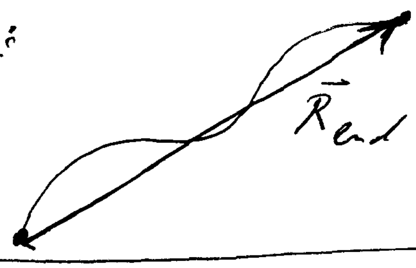
$\Psi(\vec{u}) \sim e^{-U_{bend}/k_BT} = e^{-\frac{1}{2} L_p \int_0^L \left(\frac{\partial \vec{u}}{\partial s}\right)^2 ds}$

$\Rightarrow$ Persistence length: $\boxed{L_p = \frac{k_c}{k_BT}}$

correlation length:

$$\boxed{\langle \vec{u}(s) \cdot \vec{u}(0) \rangle = e^{-s/L_p}}$$

## Kratky - Porod model:




$$\boxed{\vec{R}_{end}^2 = 2 L L_p - 2 L_p^2 \left(1 - e^{-L/L_p}\right)}$$

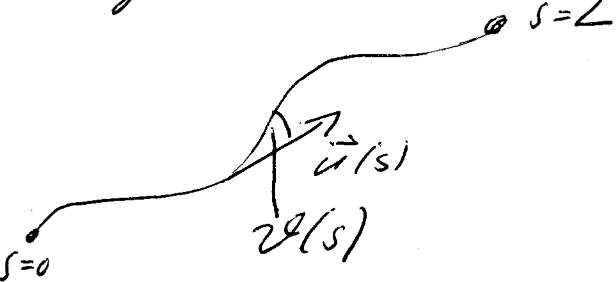
better way to measure $L_p$ by end-to-end distance see Le Goff

show transparency!

## analyzing the thermal fluctuations:

~~Fourier decomposition of u(s)~~

tangential angle $\vartheta(s)$:




$$U_{bend} = \frac{k_c}{2} \int_0^L \left(\frac{\partial \vartheta}{\partial s}\right)^2 ds$$

free filament ends: $\left.\frac{\partial \vartheta}{\partial s}\right|_{s=0,L} = 0$

Fourier expansion: $\boxed{\vartheta(s) = \sum_q a_q \cos(qs)}$

$$q = \frac{\pi n}{L} \qquad n = 1, 2, 3, .. \Rightarrow \lambda = \frac{\pi}{q}$$

$$\langle U_{bend}(q) \rangle = \frac{k_c}{2} \left\langle \int_0^L \left(\frac{\partial}{\partial s} a_q \cos(qs)\right)^2 ds \right\rangle$$

$$= \frac{k_c}{4} L q^2 \langle a_q^2 \rangle \underset{\uparrow}{=} \frac{k_B T}{2}$$

equipartion theorem

values for $L_p$:

| | | | |
|---|---|---|---|
| microtubles | actin + phalloidin | actin | vimentin |
| 1-3 mm | 16 μm | 7 μm | 300 nm |

# 3.c. Rheological Properties of Cytoskeletal Filaments

Lit.: • D. Morse et al, Viscoelasticity of tightly entangled solution of semiflexible polymers, Phys. Rev. E, 58, R1237 – R1240, 1998

• F.C. MacKintosh et al, Phys. Rev. Lett, 75 4425, 1995

• Doi and Edwards, Theory of Polymer Dynamics, Chapt. 7.3

## Linear viscoelasticity:

polymer solution is like a bowl of spaghetti: it flows for ~~[illegible]~~ slow deformations and resists elastically for fast deformations = Viscoelasticity

show actin rheology!

$$\boxed{\sigma_{\alpha\beta}(t) = \left(\kappa_{\alpha\beta}(t) + \kappa_{\beta\alpha}(t)\right)\eta}$$

viscosity

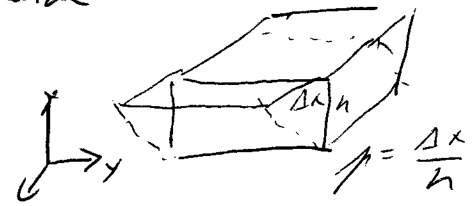
stress (pressure) tensor

velocity gradient tensor

$$\sigma_{\alpha\beta}(t) = \int_{-\infty}^{t} G(t-t')\left(\kappa_{\alpha\beta}(t') + \kappa_{\beta\alpha}(t')\right)$$

for $\kappa_{\alpha\beta}$ small justifying a linear relation

$G(t)$: shear relaxation modulus:




for shear flow: $v_x = \kappa(t)\, y \quad v_y = 0 \quad v_z = 0$

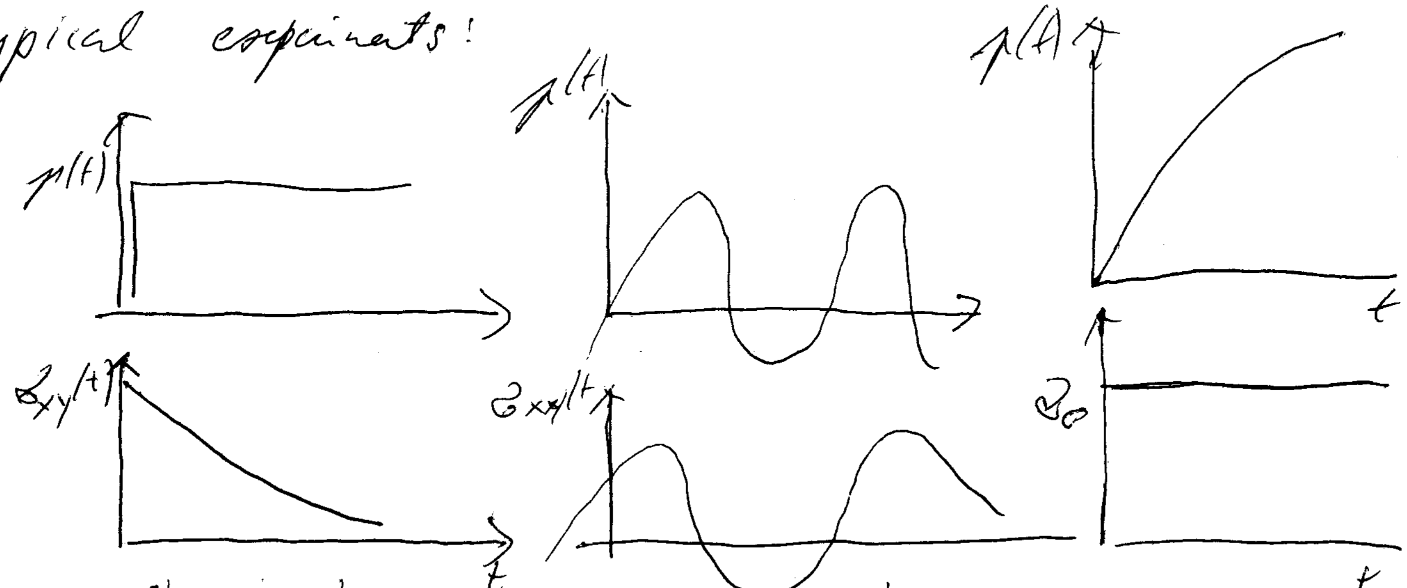
$$\Rightarrow \sigma_{xy}(t) = \int_{-\infty}^{t} dt\, G(t-t')\,\kappa(t')$$

shear strain:

$$\boxed{\gamma(t) = \int_{0}^{t} dt'\, \kappa(t')}$$

$$\Rightarrow \boxed{\sigma_{xy}(t) = \int_{-\infty}^{t} dt'\, G(t-t')\,\frac{\partial \gamma(t')}{\partial t'}}$$

typical experiments:




stepwise shear — oscillating shear — creep stepwise : $\gamma(t) = \begin{cases} 0 & (t<0) \\ \gamma_0 & (t>0) \end{cases}$

$$\sigma_{xy}(t) = G(t)\,\gamma_0$$

Oscillating :

$$\gamma(t) = \gamma_0 \cos\omega t$$

$$\Rightarrow \sigma_{xy}(t) = (\underbrace{G'(\omega)}_{\text{storage module}} \cos\omega t - \underbrace{G''(\omega)}_{\text{loss module}} \sin\omega t)\,\gamma_0$$

$$\equiv \gamma_0 \,\mathrm{Re}\left(G^*(\omega)\, e^{i\omega t}\right)$$

$$G^*(\omega) = G'(\omega) + i G''(\omega) \qquad \text{complex shear modulus}$$

$$G^*(\omega) = i\omega \int_0^\infty e^{-i\omega t} G(t)\, dt$$

Creep $\sigma(t) = \sigma_0$

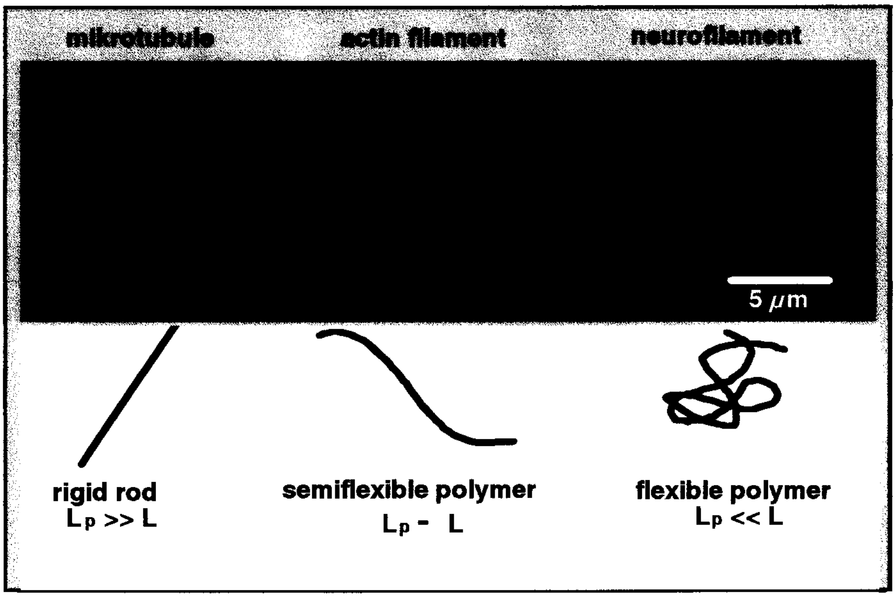
$$\gamma(t) = \sigma_0 \left( \frac{t}{g_0} + \underbrace{\frac{g_1}{g_0^2}}_{\text{steady-state compliance}} \right)$$

mit $g_0 = \underbrace{\int_0^\infty dt\, G(t)}_{= \eta_0 \text{ steady state viscosity}}$ $\quad g_1 = \int_0^\infty t\, G(t)\, dt$

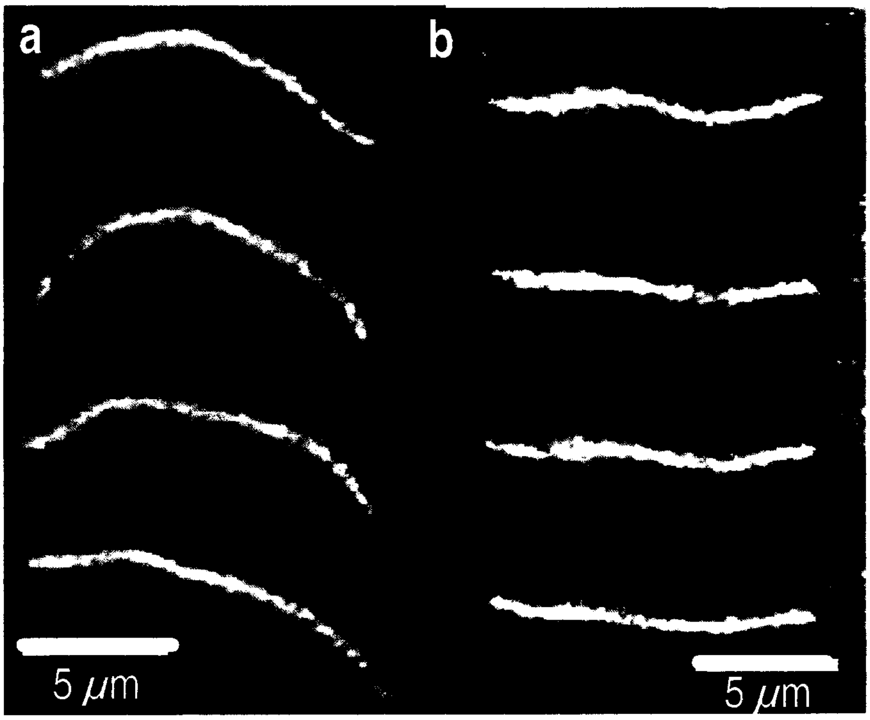
$t \gg 0$:

$$\sigma_0 = g_0 \frac{d\gamma}{dt}$$

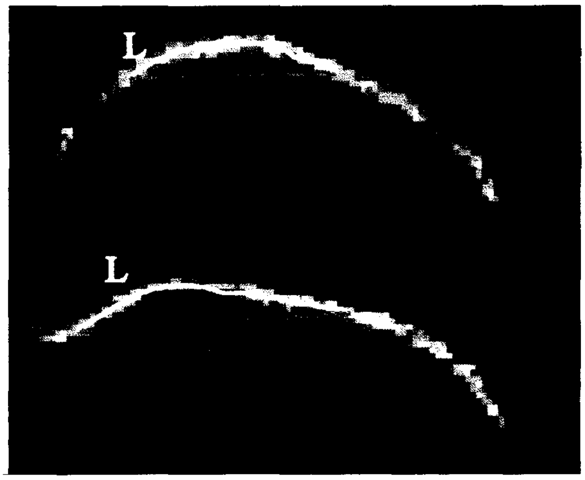
# PERSISTENCE LENGTH OF ACTIN FILAMENTS



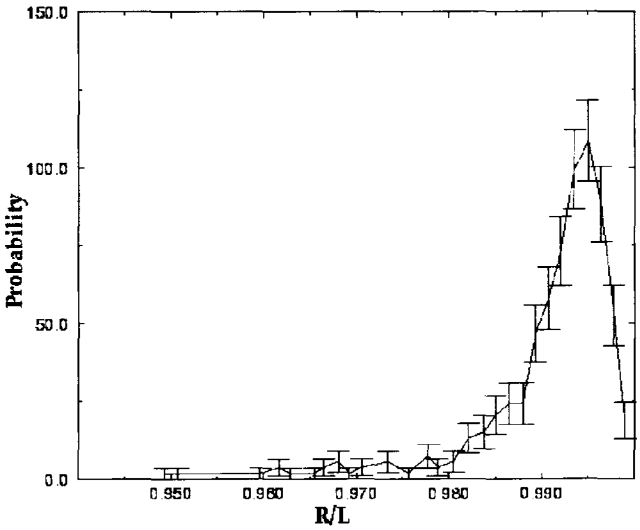

Magnitude of Brownian motions is a measure for stiffness:



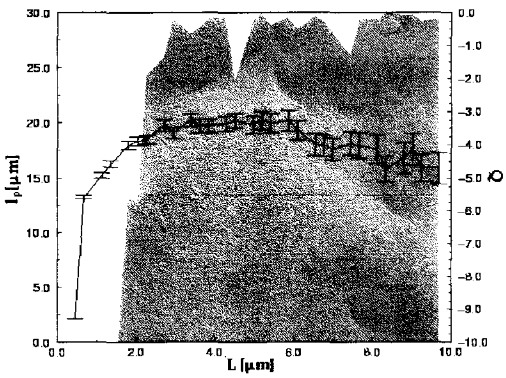
the end-to-end distance R as a function of the arc-length L determines the persistence length $L_p$:



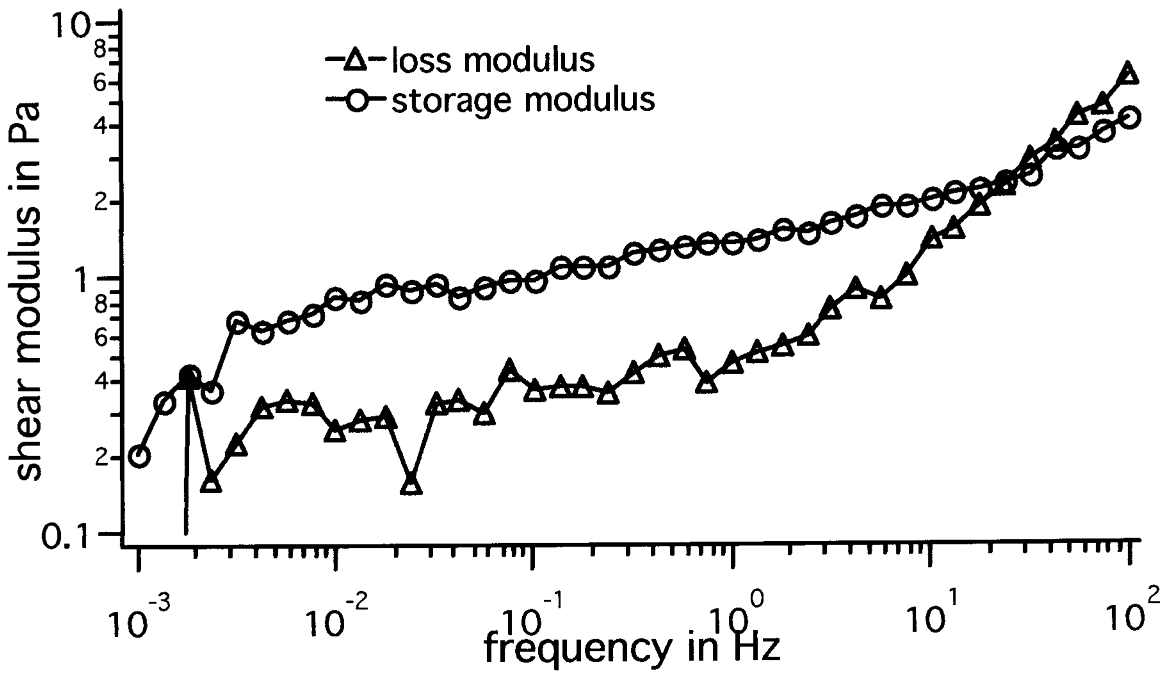

distribution of the end-to-end distance R (–> J. Wilhelm et al.) :

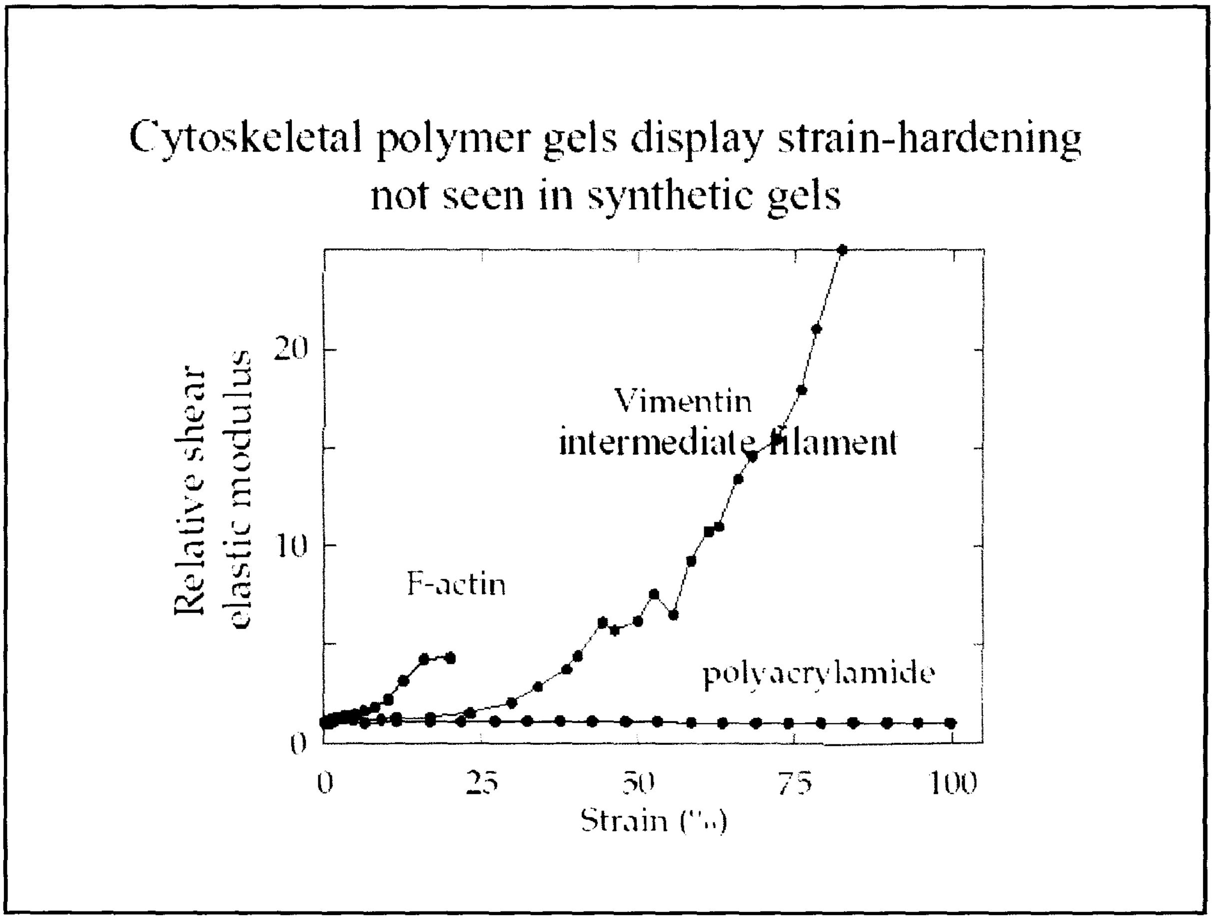



persistence length $L_p$ as a function of the wavelength :



# Viscoelastic Behavior of Actin Networks I






Strain hardening