UNIVERSITÄT LEIPZIG

Experimental Physics IV IPSP

Problem Set 5

Deadline: Thursday, 12.05.2011, before the lecture

Problem 13: 4 points

Given is the time-dependent Schrödinger Equation (SE)

$$i\hbar\frac{\partial}{\partial t}\Psi(\vec{x},t)=\widehat{H}\Psi(\vec{x},t)=\left(-\frac{\hbar^2}{2m}\nabla^2+V(\vec{x},t)\right)\Psi(\vec{x},t)\,.$$

Let V be time-independent. Use the separation-ansatz $\Psi(\vec{x},t)=\Phi(\vec{x})\,T(t)$ to obtain the time-independent (stationary) SE and calculate $T(t)$.

Hint: time-independent SE:

$$E\Phi(\vec{x})=\left(-\frac{\hbar^2}{2m}\nabla^2+V(\vec{x})\right)\Phi(\vec{x})$$

Problem 14: 4 points

Let $\Phi_1(x)$ and $\Phi_2(x)$ be solutions of the time-independent SE. Show that $\Phi_3(x)=\mathrm{c}_1\Phi_1(x)+\mathrm{c}_2\Phi_2(x)$ is also a solution of the time-independent SE.

Problem 15: 2+3+3 points

Solve the time-independent SE for a particle in a box! The potential V is given by

$$V(x)=\begin{cases}\infty & \text{for} \quad x<0\\ 0 & \text{for} \quad 0<x<d\\ \infty & \text{for} \quad x>d\end{cases}\,.$$

a) Use the ansatz $\Phi(x)=e^{-ikx}$ and the superposition principle (Problem 14) to calculate the general solution.
b) Plug in the boundary conditions for $x=0$ and $x=d$ and calculate c_2 and k.
c) Normalize the possible wavefunctions and calculate c_1:

$$\int\overline{\Phi}_\mathrm{n}\Phi_\mathrm{n}\mathrm{d}x\overset{!}{=}1$$

Hint: Look up Problem 12: $P_n(x)=\overline{\Phi}_\mathrm{n}\Phi_\mathrm{n}$.