UNIVERSITÄT LEIPZIG

Experimental Physics IV IPSP

Problem Set 6

Deadline: Thursday, 19.05.2011, before the lecture

Problem 16: 5 points

One frame of reference F_1 is moving relative to another frame of reference F_2 with a velocity v_{12}, which in turn is also moving relative to another frame of reference F_3 with a velocity v_{23}. The velocities are parallel.

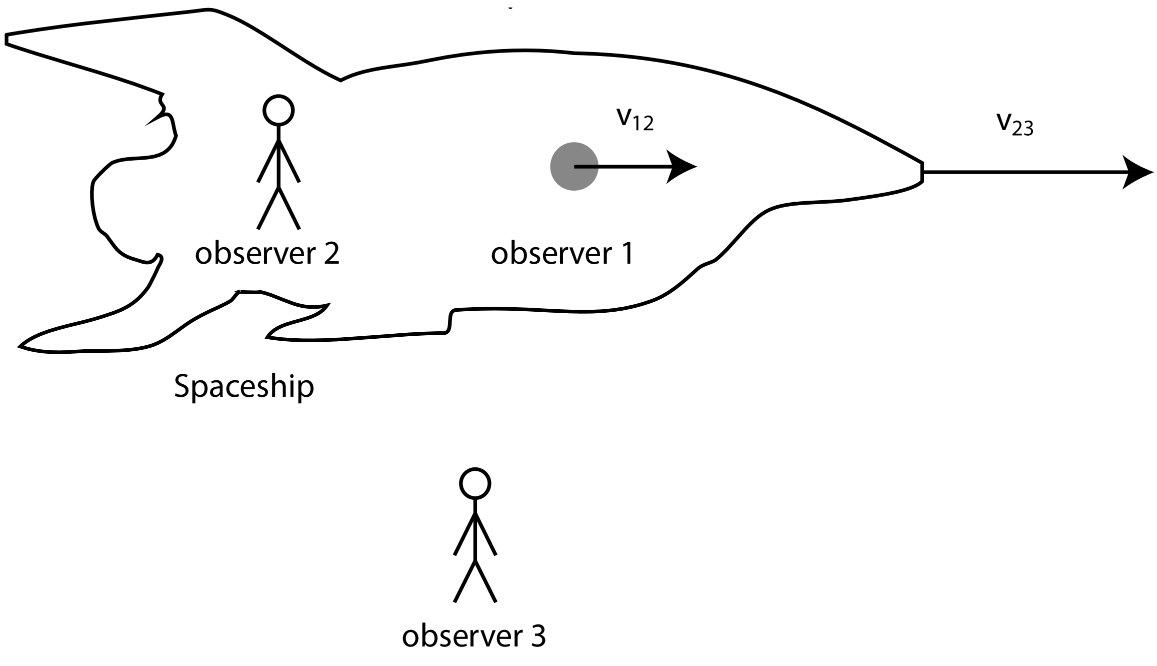
What's the relative velocity v_{13} between F_1 and F_3?

Hint: The Lorentz transformation is given by

$$t = \gamma\left(t' - \frac{v}{c^2}\,x'\right)$$
$$x = \gamma(x' - vt')$$

with $\gamma = (1 - v^2/c^2)^{-1/2}$, the elapsed time and position of the resting frame t' and x', the elapsed time and position of the moving frame t and x, and the relative velocity v between the frames.

Sketch: Throwing a ball with relativistic speed in a spaceship, which also travels with relativistic speed.



Problem 17: 2+2 points

The electromagnetic tensor is defined by

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$

with the electromagnetic four-potential $A_\mu = \eta_{\mu\nu} A^\nu = (\phi, -\vec{A}\,)$.

a) Calculate the entries of the electromagnetic tensor $F_{\mu\nu}$. *Hint:* There are only 6 independent entries.
b) The electric and magnetic field are calculated via

$$\vec{E} = (E_x, E_y, E_z) = -\frac{\partial \vec{A}}{\partial t} - \nabla\phi\,,$$
$$\vec{B} = (B_x, B_y, B_z) = \nabla \times \vec{A}\,.$$

Show that the electromagnetic field tensor can be rewritten in the following form:

$$F_{\mu\nu} = \begin{pmatrix} 0 & E_x & E_y & E_z \\ -E_x & 0 & -B_z & B_y \\ -E_y & B_z & 0 & -B_x \\ -E_z & -B_y & B_x & 0 \end{pmatrix}$$

Problem 18: 4 points

Use the equation

$$\partial_\nu F^{\mu\nu} = J^\mu\,,$$

with the four-current $J^\mu = (\rho, \vec{J}\,) = (\rho, J_x, J_y, J_z)$, to derive two Maxwell's equations

$$\nabla \cdot \vec{E} = \rho\,,$$

$$\nabla \times \vec{B} - \partial_t \vec{E} = \vec{J}\,.$$

General Hints:

$$F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu = \begin{pmatrix} 0 & -E_x & -E_y & -E_z \\ E_x & 0 & -B_z & B_y \\ E_y & B_z & 0 & -B_x \\ E_z & -B_y & B_x & 0 \end{pmatrix}$$

$$\partial_\mu = \left(\frac{d}{dt}, \frac{d}{dx}, \frac{d}{dy}, \frac{d}{dz}\right)$$

$$\partial^\mu = \left(\frac{d}{dt}, -\frac{d}{dx}, -\frac{d}{dy}, -\frac{d}{dz}\right)$$

convention: $c = 1,\ \varepsilon_0 = 1,\ \mu_0 = 1$