UNIVERSITÄT LEIPZIG

Experimental Physics IV IPSP

Problem Set 7

Deadline: Thursday, 26.05.2011, before the lecture

Problem 19: 2+1+2+1+2+2 points

The wave function of a particle in a box is given by

$$\Psi_n(x) = \sqrt{2/d}\ \sin\left(\frac{\pi n}{d}x\right).$$

Calculate:

a) $\langle n|m\rangle$
b) $\langle n|\hat{x}|n\rangle$ and $\langle n|\hat{x}^2|n\rangle$
c) $\langle n|\hat{p}|n\rangle$ and $\langle n|\hat{p}^2|n\rangle$
d) $\Delta x \Delta p$ with $\Delta f = \sqrt{\langle n|\hat{f}^2|n\rangle - \langle n|\hat{f}|n\rangle^2}$
e) $\langle n|\hat{H}|n\rangle$
f) What do you have calculated in the previous examples?

Problem 20: 5 points

The ladder operators $l^{\pm}$ for the particle in a box are defined by

$$l^{\pm}|n\rangle = |n \pm 1\rangle.$$

Derive the up- and down-ladder operator for the particle in a box.

Hint:

$$\sin(x+y) = \sin x \cos y + \cos x \sin y$$

$$\cos(ax) = \frac{1}{a}\frac{\partial}{\partial x}\sin(ax)$$

Problem 21: 3 points

In the ground state you cannot further "ladder down", therefore the down-ladder operator must fulfill the equation $l^{-}|1\rangle = 0$. Use this equation in order to obtain an ODE for the ground state and solve it.