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UNIVERSITÄT LEIPZIG

Experimental Physics IV IPSP

Problem Set 8

Deadline: Thursday, 02.06.2011, before the lecture

Problem 22: 2+3 points

The stationary wave function of the ground state of the hydrogen atom is given by

$$\Psi(r) = \frac{1}{\sqrt{\pi}\, a_0^{3/2}}\; e^{-\frac{r}{a_0}}$$

with the Bohr radius $a_0 = 4\pi\varepsilon_0\hbar^2/me^2$.

Show that this wave function is indeed a solution of the time-independent SE for the hydrogen atom.

a) Write down the SE for the hydrogen atom. Use spherical coordinates and the *Laplace operator in spherical coordinates*.
b) Plug in the wave function and energy of the ground state of hydrogen.

Problem 23: 3+1 points

The solution of the SE for a particle in a box ($0 < x < d$) is

$$\Psi_n(x) = \begin{cases} 0 & \text{for} \quad x < 0 \\ \sqrt{2/d}\,\sin\left(\frac{n\pi}{d}x\right) & \text{for} \quad 0 < x < d \\ 0 & \text{for} \quad x > d \end{cases} .$$

Translate the coordinate system so, that the new position of the box will be $-d/2 < x < +d/2$. Calculate the "new" wave functions. What is the difference between both representations of the same wave functions?
Name some possible applications for quantum dots (or nano-particles in general) for science and industries/economy (preferable applications not mentioned in the lecture).

Problem 24: 2+3+2 points

An incoming wave (coming from $-\infty$) with finite positive energy E is scattered at a potential $V(x) = -g\delta(x)$ with the delta distribution $\delta(x)$. One part of the incoming wave is reflected and the remaining is transmitted. Therefore, the general solution is

$$\Psi = \begin{cases} e^{ikx} + re^{-ikx} & \text{for} \quad x < 0 \\ te^{ikx} & \text{for} \quad x > 0 \end{cases}$$

with the wave vector $k = \sqrt{2mE}/\hbar$ and $1 + r = t$

Calculate the Reflection and Transmission coeeficient $R = |r|^2$ and $T = |t|^2$.

a) Use your knowledge about the delta-distribution to verify the equation for the boundary condition:

$$\partial_x \Psi(0^-) - \partial_x \Psi(0^+) = \frac{2mg}{\hbar^2} \Psi(0)$$

b) Calculate r and t using the boundary condition above.

c) Finally, calculate R and T. Draw a sketch of the energy-dependent Reflection and transmission coefficient $R(E)$ and $T(E)$.