UNIVERSITÄT LEIPZIG

Experimental Physics IV IPSP

Problem Set 10

Deadline: Thursday, 16.06.2011, before the lecture

Problem 27: 2+2+1 points

The Lennard-Jones potential

$$V(r) = \varepsilon\left(\left(\frac{r_0}{r}\right)^{12} - 2\left(\frac{r_0}{r}\right)^{6}\right)$$

is a basic model to describe the interaction of two non-charged atoms.

a) Calculate the position $r_{\min}$and the depth $V_{\min}$of the minimum of the potential
b) Derive the Taylor series at the minimum $r_{\min}$ up to the second order (harmonic oscillator order) to obtain an approximate potential.
c) Draw a sketch of both potentials.

Problem 28: 1+2+2 points

The SE of the harmonic oscillator is given by

$$\hat{H} = \frac{\hat{p}^2}{2m} + \frac{1}{2}m\omega^2\hat{x}^2 \ .$$

The position and momentum operator can be expressed by

$$\hat{x} = x_0 + \delta x \quad \text{with} \quad \sqrt{\langle(\delta x)^2\rangle} = \Delta x$$

$$\hat{p} = p_0 + \delta p \quad \text{with} \quad \sqrt{\langle(\delta p)^2\rangle} = \Delta p$$

with the mean position $x_0 = \langle\Psi|\hat{x}|\Psi\rangle$ and mean momentum $p_0 = \langle\Psi|\hat{p}|\Psi\rangle$ and a fluctuation of the position δx and momentum δp around the mean value with the standard deviation Δx and Δp.

Use the representation of the position and momentum and Heisenberg's uncertainty principle to calculate the energy for the ground state.

a) Utilize the symmetry of the system to obtain x_0 and p_0.
b) Derive the Hamiltonian $\langle\hat{H}\rangle$ as a function of Δp ***or*** Δx by plugging in $\hat{x}$ and $\hat{p}$ and Heisenberg's uncertainty principle.

c) The minimum average energy of the harmonic oscillator is the ground state. Therefore, it has to fulfill

$$\frac{\partial\langle\hat{H}\rangle}{\partial(\Delta p)} = \frac{\partial\langle\hat{H}\rangle}{\partial(\Delta x)} = 0\,.$$

Problem 29: 1+1+1+1+1+1+1 points

Let A, B and C be operators. The commutator is defined as

$$[A, B] = AB - BA\,.$$

Calculate the following commutators:

a) $[p_x, x]$ and $[p, r]$ with the momentum operator $p = (p_x, p_y, p_z)$ and position operator $r = (x, y, z)$, (See lecture notes)
b) $[x, y]$, $[p_y, z]$ and $[p_z, p_x]$.

Show the the following commutator relations:

c) $[A, B + C] = [A, B] + [A, C]$,
d) $[A, BC] = [A, B]C + B[A, C]$.

The angular momentum operator $L = (L_x, L_y, L_z)$ is defined by

$$L = r \times p\,.$$

Calculate the following commutators:

e) $[x, L_x]$ and $[p_x, L_x]$,
f) $[L_x, L_y]$,
g) $[L_x, L^2]$. *Hint:* "$\mathrm{c} + \mathrm{d} + \mathrm{f} = \mathrm{g}$"