UNIVERSITÄT LEIPZIG

Experimental Physics IV IPSP

Problem Set 11

Deadline: Thursday, 23.06.2011, before the lecture

Problem 30: 1+1+1+2 points

Given are the ladder operators of the harmonic oscillator

$$a^\dagger = \frac{1}{\sqrt{2}}\left(\sqrt{m\omega/\hbar}\, x - i\sqrt{1/m\omega\hbar}\, p\right),$$
$$a = \frac{1}{\sqrt{2}}\left(\sqrt{m\omega/\hbar}\, x + i\sqrt{1/m\omega\hbar}\, p\right)$$

with the number operator $\hat{N} = a^\dagger a$.

Calculate the following commutators:

a) $[N, a]$ and $[N, a^\dagger]$
b) $[a, H]$ and $[a^\dagger, H]$
c) $\left[a, \left(a^\dagger\right)^n\right]$, $n \in \mathbb{N}$
d) $\left[a, e^{(a^\dagger)}\right]$ with $e^{(a^\dagger)} = \sum_{n=0}^{\infty} \frac{1}{n!} \left(a^\dagger\right)^n$

Problem 31: 5 points

Given is the potential of the one-sided harmonic oscillator

$$V(x) = \begin{cases} +\infty & \text{for} \quad x < 0 \\ \frac{1}{2} m\omega^2 x^2 & \text{for} \quad x \geq 0 \end{cases}.$$

"Calculate" the energy levels E_n and normalized wave function Ψ_n.

Hints:

- Wave function and energy levels of the normal harmonic oscillator ($\alpha = \sqrt{m\omega/\hbar}$):

$$\Psi_n(x) = \left(\frac{\alpha^2}{\pi}\right)^{\frac{1}{4}} \frac{1}{\sqrt{2^n n!}} H_n(\alpha x) \exp\left(-\frac{1}{2}(\alpha x)^2\right),$$
$$E_n = \hbar\omega\left(n + \frac{1}{2}\right)$$

- Boundary conditions of the particle in a box