UNIVERSITÄT LEIPZIG

Experimental Physics IV IPSP

Problem Set 4

Deadline: Thursday, 09.05.2011, before the seminar

Problem 11: 4+3 points

Two observers move towards each other. One sends a signal to the receiver. Classically (signal = sound), the formula for the Doppler-shifted frequency is given by

$$f_D = \frac{c + v_D}{c - v_S}\, f_S$$

with the propagation speed of the signal c, the velocity of the detector v_D, the velocity of the source v_S, the emitted frequency of the source f_s and the detected frequency f_D.
Furthermore, in the equivalent case of a relativistic signal (e.g. light, EM wave) the time dilation, which describes the difference of elapsed time between two relative moving observers, is given by

$$\Delta T' = \frac{\Delta T}{\sqrt{1 - v^2/c^2}}$$

with the elapsed time $\Delta T'$ in the reference frame of the detector and the elapsed time ΔT in the reference frame of the source with the relative velocity v.

a) Show that the relativistic Doppler-shifted frequency can be calculated to

$$f_{D,\mathrm{rel}} = \frac{\sqrt{1 + v/c}}{\sqrt{1 - v/c}} f_S$$

depending only on the relative speed v. Compare it to the classical Doppler-effect. What is special?

b) How fast do you have to drive in order to observe a red ($\lambda_{\mathrm{red}} = 670\ \mathrm{nm}$) traffic light "switching" to green ($\lambda_{\mathrm{green}} = 547\ \mathrm{nm}$) due to the relativistic Doppler-effect?

Hint: Einstein's postulate in *On the electrodynamics of Moving Bodies*: The speed of light is constant in all inertial systems, regardless how they move relative to each other. Therefore, Use an appropriate frame of reference.

Problem 12: 6 points

Protons and electrons are accelerated in a linear accelerator with a voltage of 3 MV. The relativistic energy is given by $E^2 = m_0^2 c^4 + p^2 c^2$ with the relativistic momentum $= m_0\, v\, (1 - v^2/c^2\,)^{-1/2}$.

Calculate the relativistic *and* non-relativistic velocity of both the protons and electrons in units of c. Compare the relativistic and non-relativistic velocities. How big is the difference of the relativistic effects? Are the non-relativistic calculations justified? *Hint*: E is the *total* energy of the particle.

Mass of the proton: $m_P = 938 \frac{\mathrm{MeV}}{\mathrm{c}^2}$

Mass of the electron: $m_e = 0.511 \frac{\mathrm{MeV}}{\mathrm{c}^2}$

Problem 13: 6+1 points

The probability distribution of a particle in a one-dimensional box (dimension of the box: $0 < x < d$) is given by

$$P_{\text{classical}}(x) = \frac{1}{d}\,,$$

$$P_{n,\text{quantum}}(x) = \frac{2}{d}\sin^2\left(\frac{n\pi x}{d}\right)\,, \quad n = 1,2,3,\ldots$$

for a classical and quantum-mechanical particle, respectively.

a) Calculate the expectation values $\langle x \rangle$ and $\langle x^2 \rangle$ for the classical and the quantum-mechanical particle in a box.
b) What happens in the limit case of increasing n and what does it mean?

Hint:

$$\int \sin^2 x \, \mathrm{d}x = \frac{x}{2} - \frac{1}{4}\sin 2x\,,$$

$$\int x \sin^2 x \, \mathrm{d}x = \frac{x^2}{4} - \frac{x}{4}\sin 2x - \frac{1}{8}\cos 2x\,,$$

$$\int x^2 \sin^2 x \, \mathrm{d}x = \frac{x^3}{6} - \left(\frac{x^2}{4} - \frac{1}{8}\right)\sin 2x - \frac{x}{4}\cos 2x\,.$$

Problem 14: 3 points

Calculate the propability of the occurrence of Balmer lines in the hydrogen spectrum of the sun (T= 6000K). Hints: Consider the degeneration of energy states. How are the occupied states distributed?