Universität Leipzig, Fakultät für Physik und Geowissenschaften

Exercises for Experimental Physics 4 – IPSP

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Exercise Sheet 7 (Summer Term 2013)

Date of Issue to Students: May 28^{th} 2013

Date of Submission: June 4^{th} 2013

Submission Place: Marked mailbox next to room 302 (Linnestr. 5)
Submission Time: 11:00 a.m. at the submission day noted above

Please note: Write your name and matriculation number on EACH sheet of paper.
Only submit the calculations and results for exercise 1-3, exercise 4 will be discussed during the instruction classes.

Exercises:

1. (a) Given is the time-dependent Schrödinger Equation (SE)
   $i\hbar\frac{\partial}{\partial t}\Psi(\vec{x},t) = \left(\frac{-\hbar^2}{2m}\nabla^2 + V(\vec{x},t)\right)\Psi(\vec{x},t)$.

   Let V be time-independent. Use the separation-ansatz $\Psi(\vec{x},t) = \Phi(\vec{x})T(t)$ to obtain the time-independent (stationary) SE and calculate T(t). *Hint*: $f(\vec{x}) = g(t) = const$. Look up the solution for a free particle in order to determine the physical meaning of the freely choosen constant. (4 Points)

   (b) Let $\Phi_1(x)$ and $\Phi_2(x)$ be solutions of the stationary SE. Show that $\Phi_3(x) = c_1\Phi_1(x) + c_2\Phi_2(x)$ is also a solution of the stationary SE. (4 Points)

2. The stationary wavefunction of the ground state of the hydrogen atom is $\Psi(r) = \frac{1}{\sqrt{\pi}a_0^{3/2}}e^{\frac{-r}{a_0}}$ with the Bohr radius $a_0 = 4\pi\epsilon_0\hbar^2/me^2$. Show that this wavefunction is indeed a solution of the stationary SE of the hydrogen atom. (a) Write down the SE for the hydrogen atom. Use spherical coordinates and the *Laplace operator in spherical coordinates*. (2 Points) (b) Plug in the wave function and energy of the ground state of hydrogen. (3 Points)

3. The probability distribution of a particle in a one-dimensional box (dimension: $0 < x < d$) is given by $P_{classical}(x) = 1/d$ and $P_{n,quant}(x) = |\Psi(x)|^2 = \frac{2}{d}\sin^2(\frac{n\pi x}{d})$ with $n = 1, 2, 3, ...$ for a classical and a quantum-mechanical particle, respectively.

   (a) Calculate the expectation values $\langle x\rangle$ and $\left\langle x^2\right\rangle$ for the classical and the quantum-mechanical particle in a box. (5 Points)

   (b)What happens in the limit case of increasing n? (2 Points)

   Hint:
   $\int \sin^2(x)\mathrm{d}x = \frac{x}{2} - \frac{1}{4}\sin(2x)$,
   $\int x\sin^2(x)\mathrm{d}x = \frac{x^2}{4} - \frac{x}{4}\sin(2x) - \frac{1}{8}\cos(2x)$,
   $\int x^2\sin^2(x)\mathrm{d}x = \frac{x^3}{6} - \left(\frac{x^2}{4} - \frac{1}{8}\right)\sin(2x) - \frac{x}{4}\cos(2x)$

4. An exited state of an electron decays exponentially with a certain life time τ by emitting a photon: $P(t) = e^{-|t|/\tau}$. This uncertainty in time leads to an uncertainty of the energy of the emitted photon.

   (a) Calculate the shape of the emission spectrum of the photon by using Fouriertransformation $\tilde{P}(\omega) = FT[P(t)](\omega)$

   (b) Calculate the half width frequency $\omega_{1/2}$ where the intensity of the emsission spectrum drops to $\frac{1}{2}\tilde{P}_{\text{MAX}}$