

# Diffusion in Oxides – here: Oxygen Ion Conductors

M. Martin  
Institute of Physical Chemistry  
RWTH Aachen University  
Germany

R.A. De Souza, D. Samuelis, O. Schulz

I.V. Belova, G.E. Murch  
Diffusion in Solids Group  
The University of Newcastle  
Australia

# Walther Nernst

## Nernst Glower

Deutsches Patent

Nr. 104872 vom 6.7.1897



filament

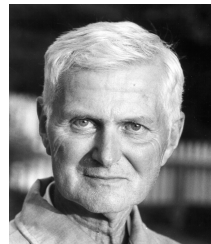
Filament  
(Nernstmasse)



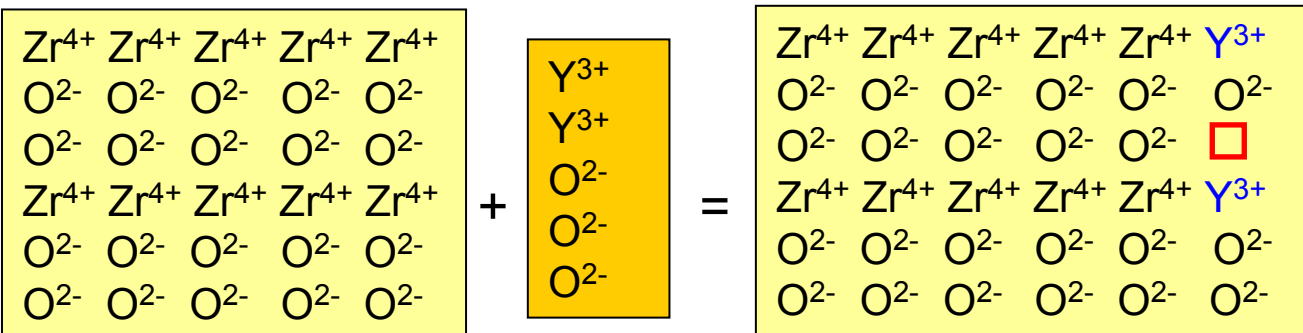
Chalk, magnesia,  
zirconia, rare  
earths



# Carl Wagner



Über den Mechanismus der elektrischen Stromleitung im Nernststift,  
*Naturwissenschaften* 31 (1943) 265.

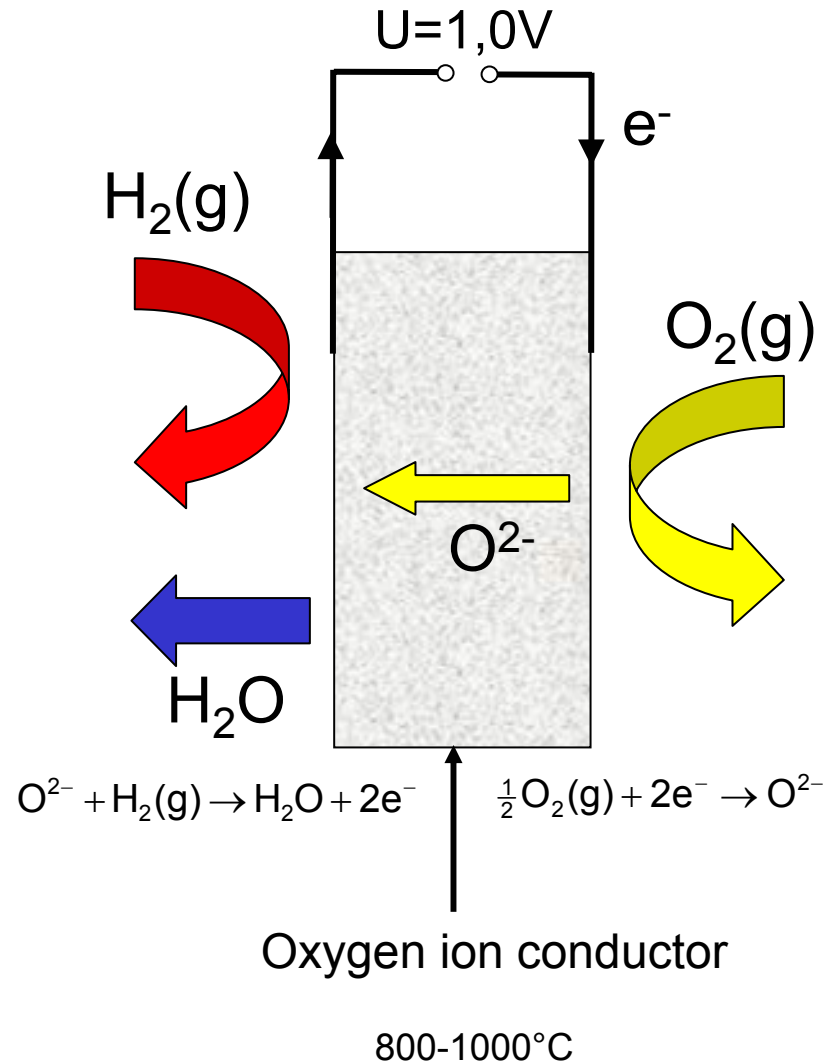


$[Y'_{Zr}] = 2[V_{\ddot{O}}]$   
Majority defects

Vacancies,  
in Rome already known 27 BC



# Solid Oxide Fuel Cell (SOFC)



# Ionic conductivities of oxide electrolytes: $\sigma \propto D_V \cdot c_V$

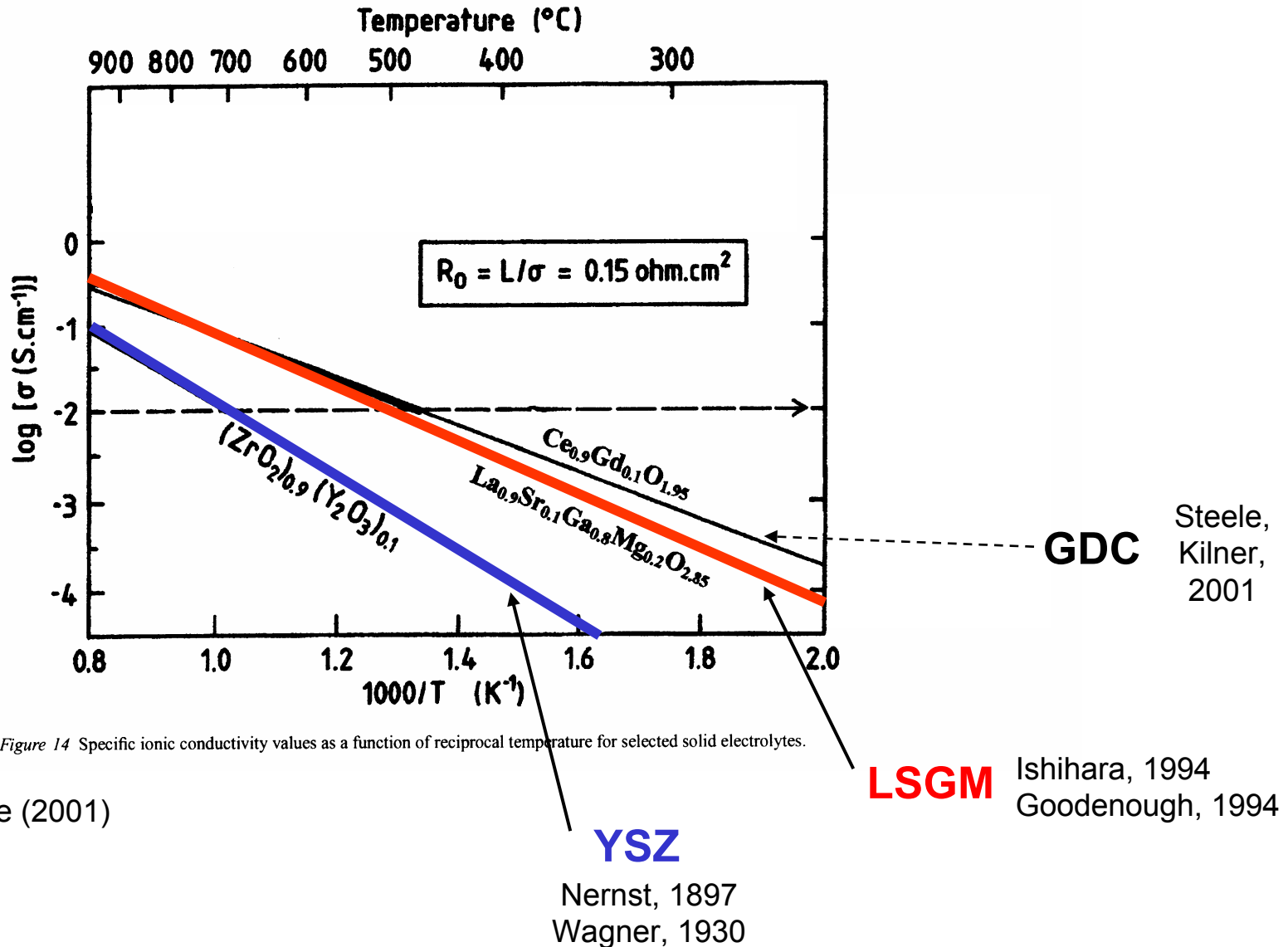


Figure 14 Specific ionic conductivity values as a function of reciprocal temperature for selected solid electrolytes.

B.C.H. Steele (2001)

**YSZ**

Nernst, 1897  
Wagner, 1930

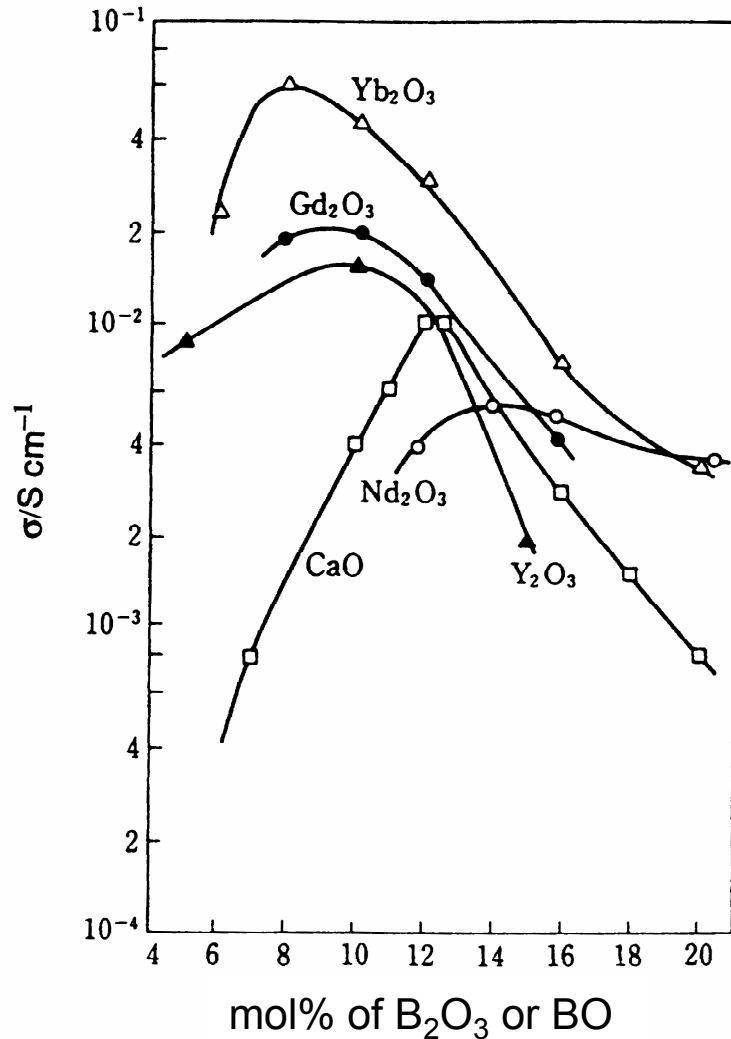
**LSGM**

Ishihara, 1994  
Goodenough, 1994

**GDC**

Steele,  
Kilner,  
2001

# Ionic conductivities of doped $\text{ZrO}_2$ : $\sigma \propto D_V \cdot c_V$



H. Tannenberger (1965)

## Maximum due to:

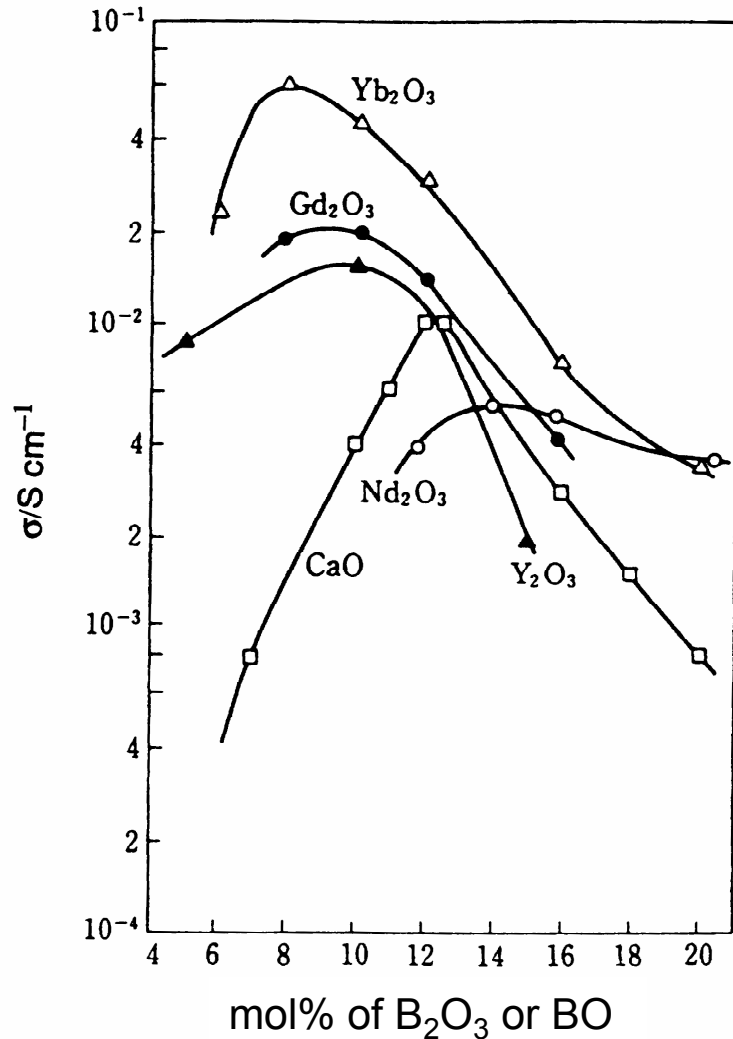
- defect clusters
- defect ordering



number of mobile oxygen vacancies decreases

- H. Schmalzried, Z. Phys. Chem. NF, 105, 47 (1977)
- A.D. Murray, G.E. Murch, C.R.A. Catlow, Solid State Ionics, 19, 196 (1986)
- F. Shimojo, T. Okabe, F. Tachibana, M. Kobayashi, H. Okazaki, J. Phys. Soc. Japan, 61, 2842 (1992)
- M. Meyer, N. Nicoloso, Ber. Bunsenges. Phys. Chem., 101, 1393 (1997)
- M.S. Khan, M.S. Islam, D.R. Bates, J. Mater. Chem., 8, 2229 (1998)
- M.O. Zacate, L. Minervini, D.J. Bradfield, R.W. Grimes, K.E. Sickafus, Solid State Ionics, 128, 243 (2000)
- R. Krishnamurthy, Y.-G. Yoon, D.J. Srolovitz, R. Carr, J. Am. Cer. Soc., 87, 1821 (2004)
- T. Ishii, T. Ishikawa, Solid State Ionics (2006)
- ...

# Ionic conductivities of doped $\text{ZrO}_2$ : $\sigma \propto D_V \cdot c_V$

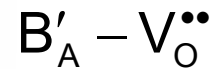


## Maximum due to:

- defect clusters
- defect ordering



number of mobile oxygen vacancies decreases



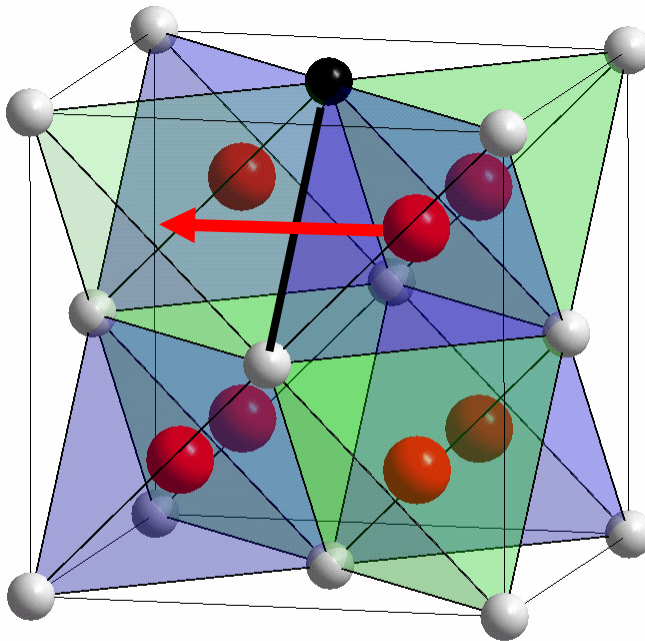
- undersized cations: nn
- oversized cations: nnn

**Cluster at 10 – 20 % dopant level ?**

# YSZ (fluorite structure)

Shimojo et al. (1992): MD

Krishnamurthy et al. (2004): DFT



Edge	$E_{AB} / \text{eV}$
Zr - Zr	0.58
Zr - Y	1.29
Y - Y	1.86

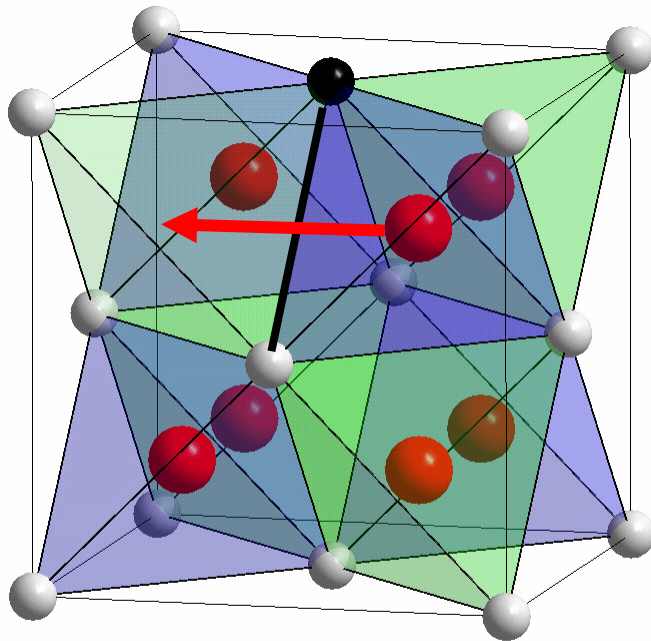
Y - Y edges are „blocking“

→ maximum in  $\sigma_{\text{oxygen}}$





## YSZ (fluorite structure)



Shimojo et al. (1992): MD

Krishnamurthy et al. (2004): DFT

Edge	$E_{AB} / \text{eV}$
Zr - Zr	0.58
Zr - Y	1.29
Y - Y	1.86

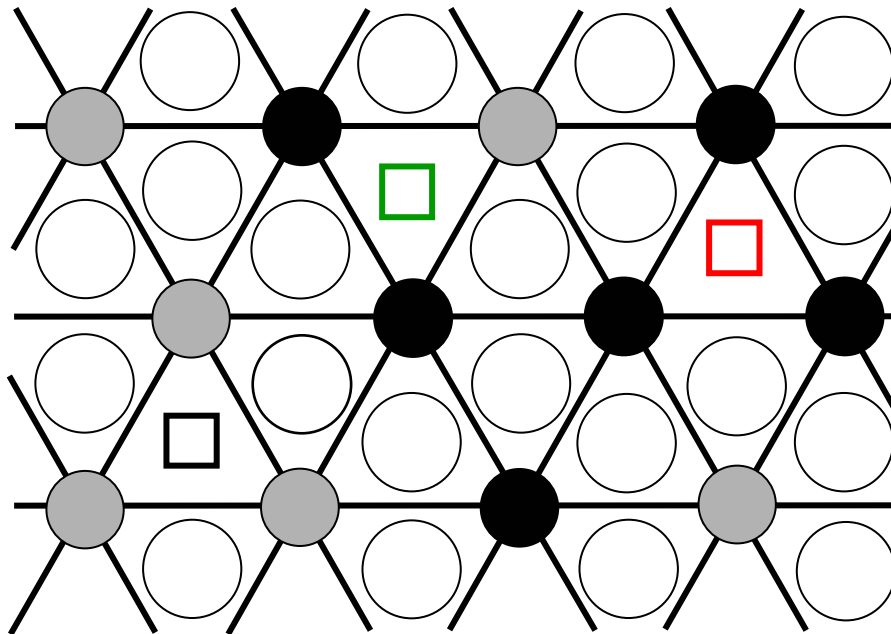
Y - Y edges are „blocking“

→ maximum in  $\sigma_{\text{oxygen}}$

But: **no Y- $V_{\text{O}}$  interactions**

**random vacancy distribution**

## Concentrated solution of $(A_A, B_A)$ and $(O_O, V_O)$



Only nearest neighbor interactions  $B_A-V_O$

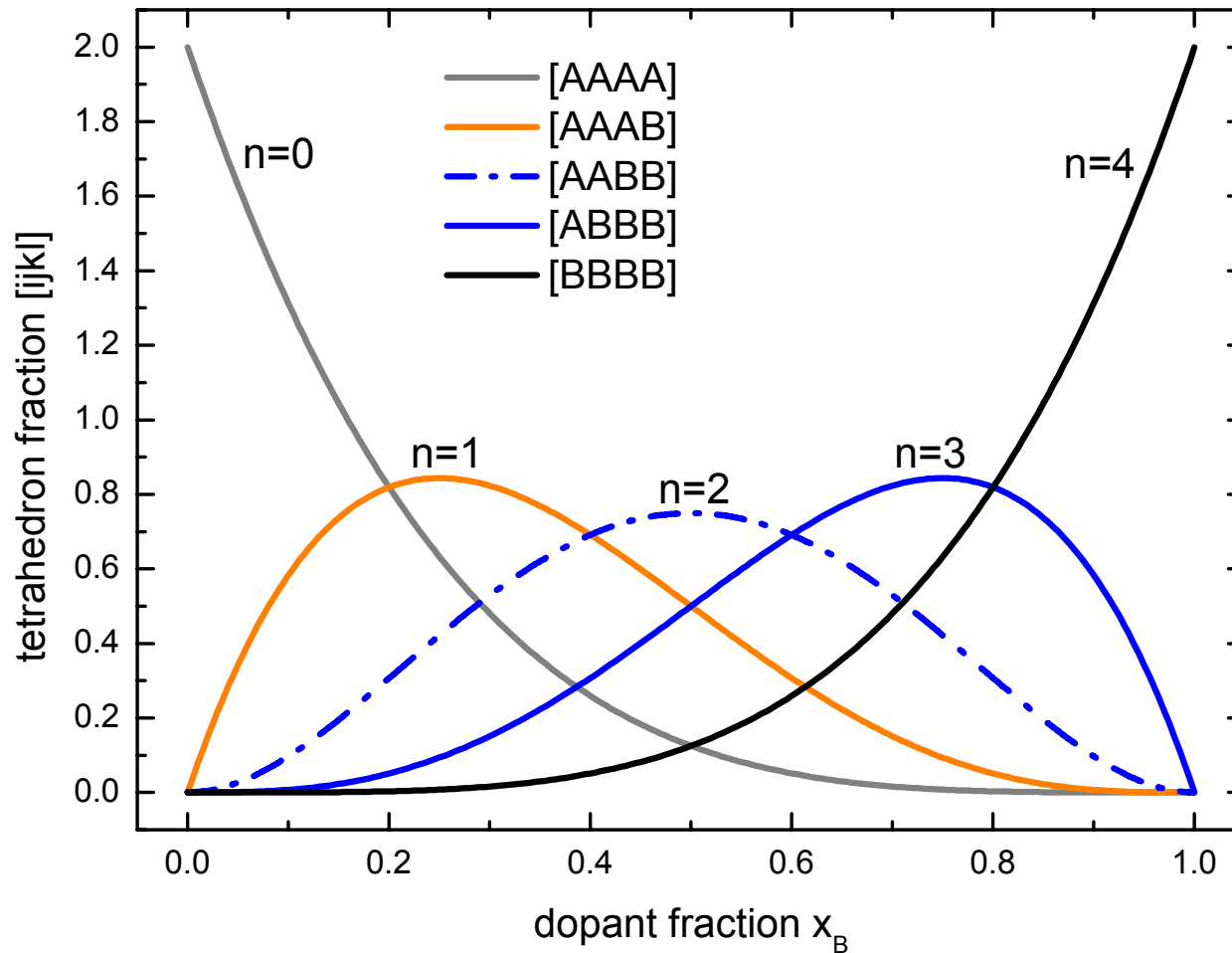
### Simple model for the oxygen ion conductivity:

1. Random cation distribution, immobile cations
2. Oxygen vacancy distribution (random, non-random)
3. Vacancy jump types
4. Ionic conductivity

# 1. Random cation distribution

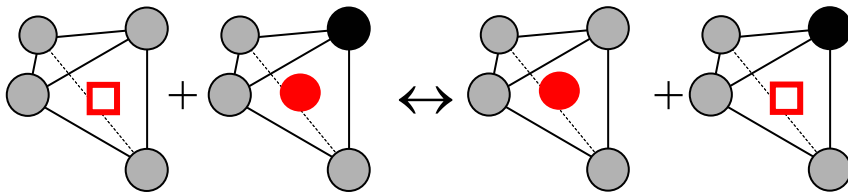
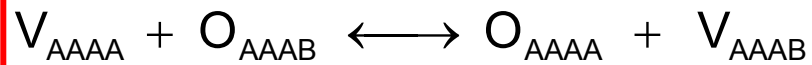
$$f_n(x_B) = 2 \binom{4}{n} x_B^n (1-x_B)^{4-n}$$

fraction of  
tetrahedra  
with n  
B-cations



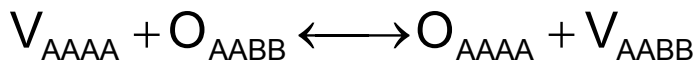
2. Vacancy distribution:  $(\text{Zr}_{1-y}^{4+} \text{Y}_y^{3+})\text{O}_{2-x/2}^{2-}$  or  $(\text{Zr}_{1-y}^x \text{Y}'_y)(\text{O}_{2-y/2}^x \text{V}_{y/2}^{\bullet\bullet})$

attractive



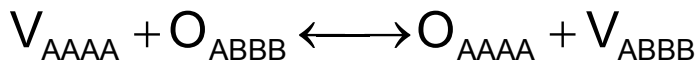
$$K_1 = \frac{[\text{O}_{\text{AAAA}}] \cdot [V_{\text{AAAB}}]}{[V_{\text{AAAA}}] \cdot [\text{O}_{\text{AAAB}}]}$$

$$K_1 = \exp\left(-\frac{\Delta E_1}{kT}\right)$$

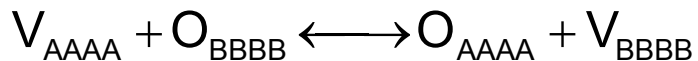


$$K_2 = \frac{[\text{O}_{\text{AAAA}}] \cdot [V_{\text{AABB}}]}{[V_{\text{AAAA}}] \cdot [\text{O}_{\text{AABB}}]}$$

$$K_2 = \exp\left(-\frac{\Delta E_2}{kT}\right)$$



...

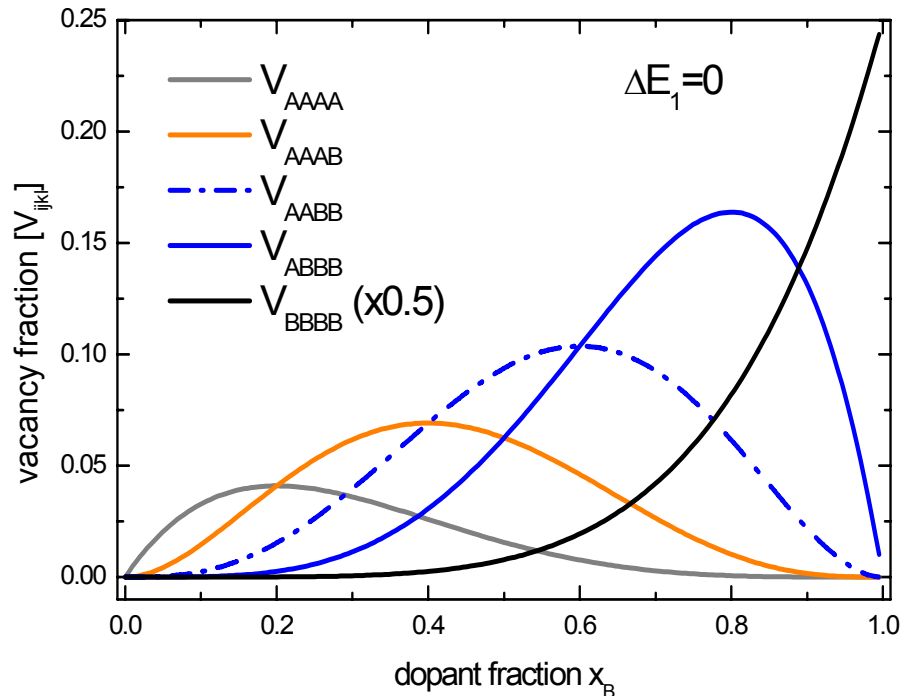


...

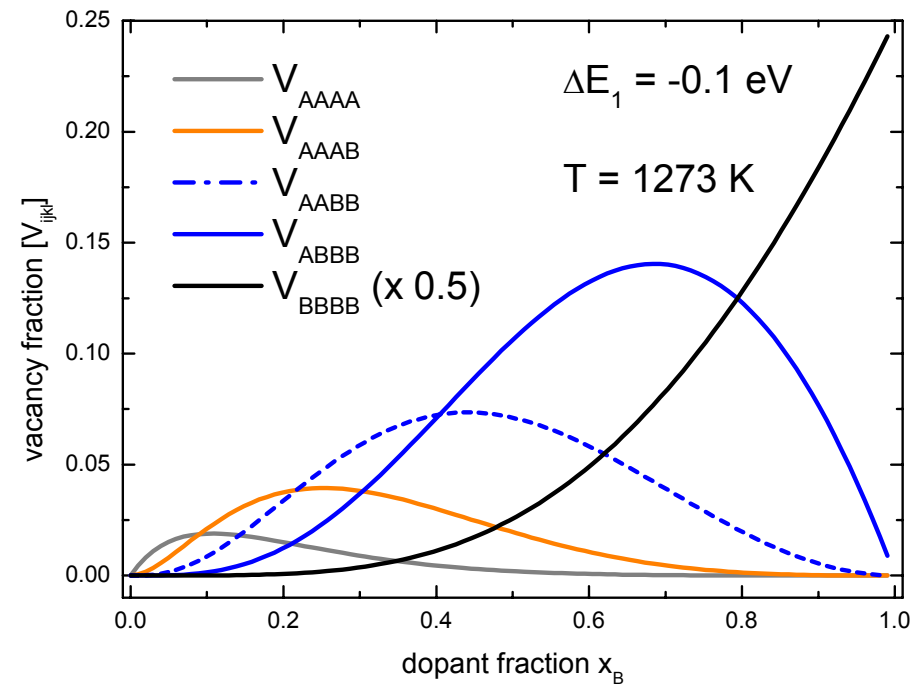
Charge neutrality

$$x_B = 2([\text{V}_{\text{AAAA}}] + [\text{V}_{\text{AAAB}}] + [\text{V}_{\text{AABB}}] + [\text{V}_{\text{ABBB}}] + [\text{V}_{\text{BBBB}}])$$

- no B-V interaction
- random vacancy distribution



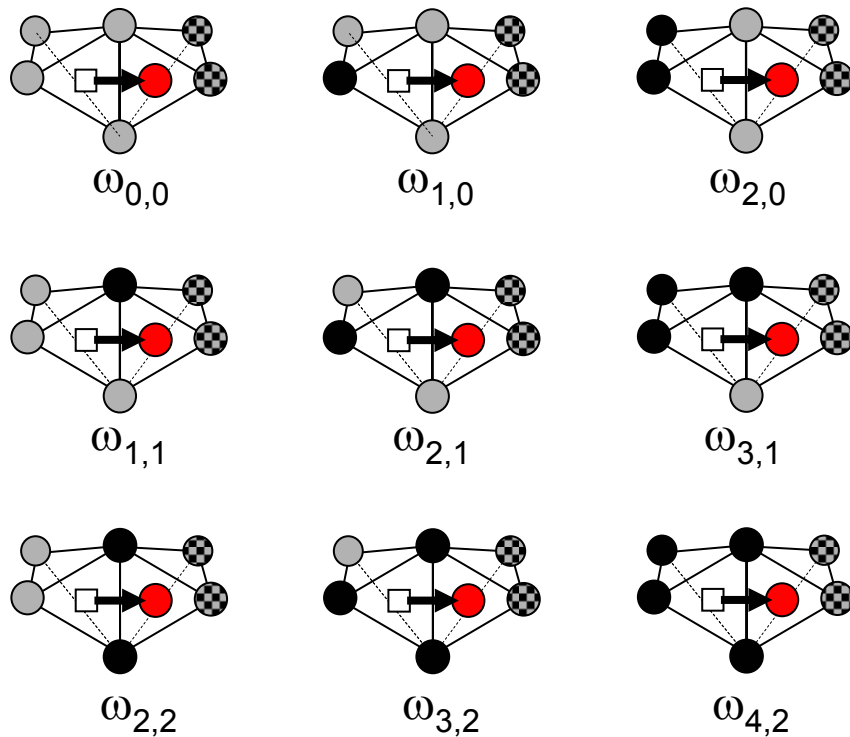
- attractive B-V interaction
- non-random vacancy distribution



more vacancies in  
B-containing tetrahedra

$$\Delta E_n = n \cdot \Delta E_1$$

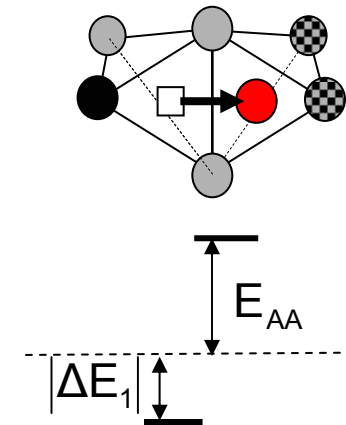
### 3. Jump types



### Site energies

$$\Delta E_n = n \cdot \Delta E_1$$

$$n = 0, 1, 2, 3, 4$$

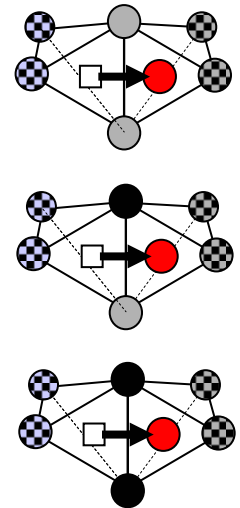
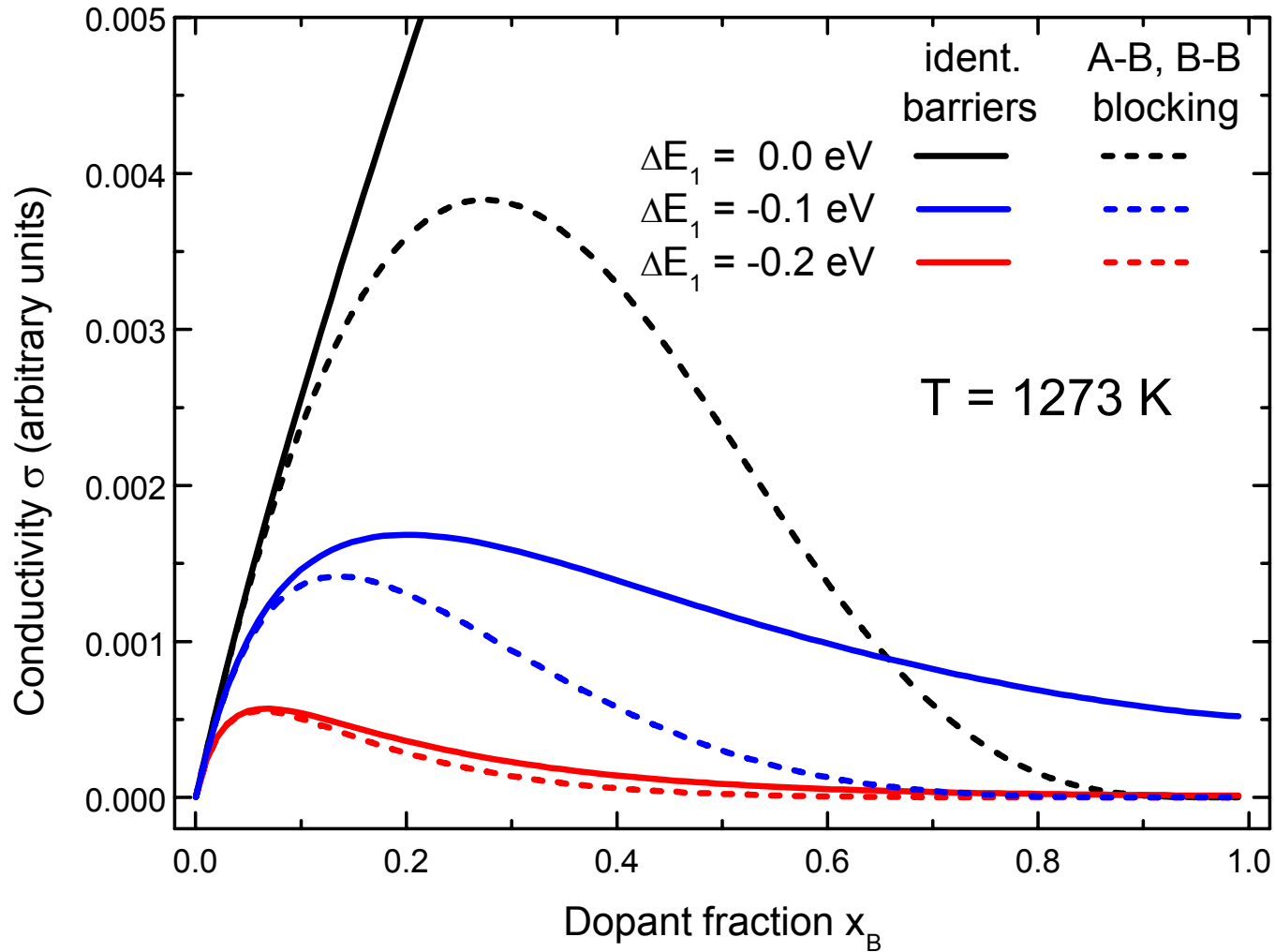


### 4. Oxygen ion conductivity

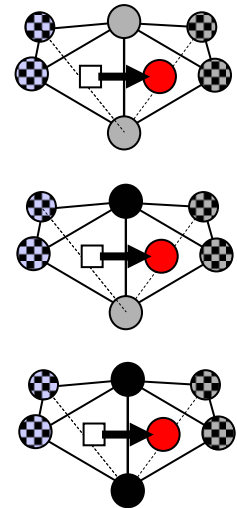
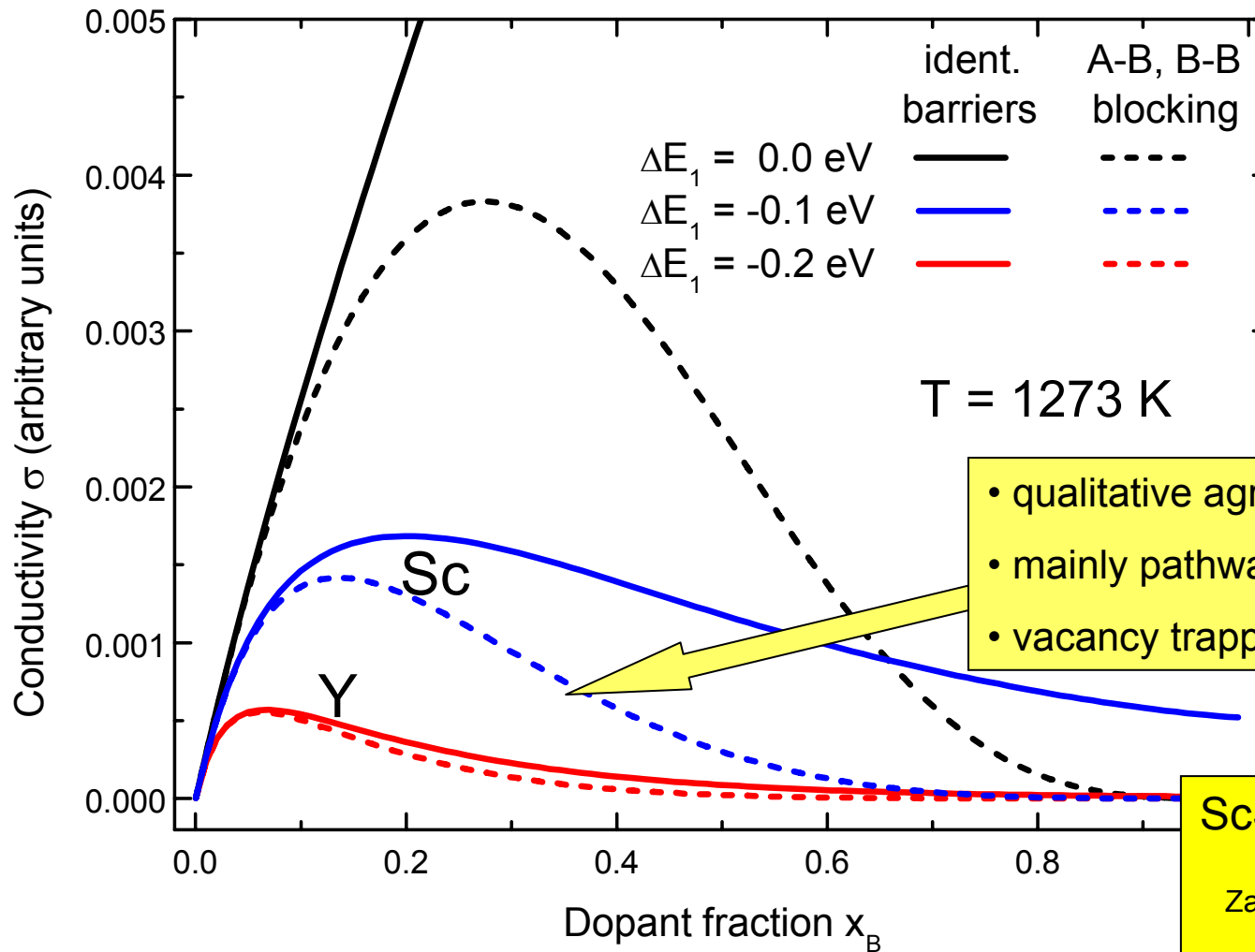
$$\sigma_{\infty} \left( [V_{AAAA}] \cdot 6 \cdot \omega_{0,0} \cdot ([O_{AAAA}] + [O_{AAAB}] + [O_{AABB}]) \right. \\
+ [V_{AAAB}] \cdot \left\{ 3 \cdot \omega_{1,0} \cdot ([O_{AAAA}] + [O_{AAAB}] + [O_{AABB}]) + 3 \cdot \omega_{1,1} \cdot ([O_{AAAB}] + [O_{AABB}] + [O_{ABBB}]) \right\} \\
+ [V_{AABB}] \cdot \left\{ + \omega_{2,0} \cdot ([O_{AAAA}] + [O_{AAAB}] + [O_{AABB}]) + \omega_{2,2} \cdot ([O_{AABB}] + [O_{ABBB}] + [O_{BBBB}]) \right. \\
\left. + 4 \omega_{2,1} \cdot ([O_{AAAB}] + [O_{AABB}] + [O_{ABBB}]) \right\} \\
+ [V_{ABBB}] \cdot \left\{ 3 \cdot \omega_{3,2} \cdot ([O_{AABB}] + [O_{ABBB}] + [O_{BBBB}]) + 3 \cdot \omega_{3,1} \cdot ([O_{AAAB}] + [O_{AABB}] + [O_{ABBB}]) \right\} \\
\left. + [V_{BBBB}] \cdot 6 \cdot \omega_{4,2} \cdot ([O_{AABB}] + [O_{ABBB}] + [O_{BBBB}]) \right)$$

Correlated motion

# Ionic conductivity



# Ionic conductivity





# Ionic conductivities of oxide electrolytes

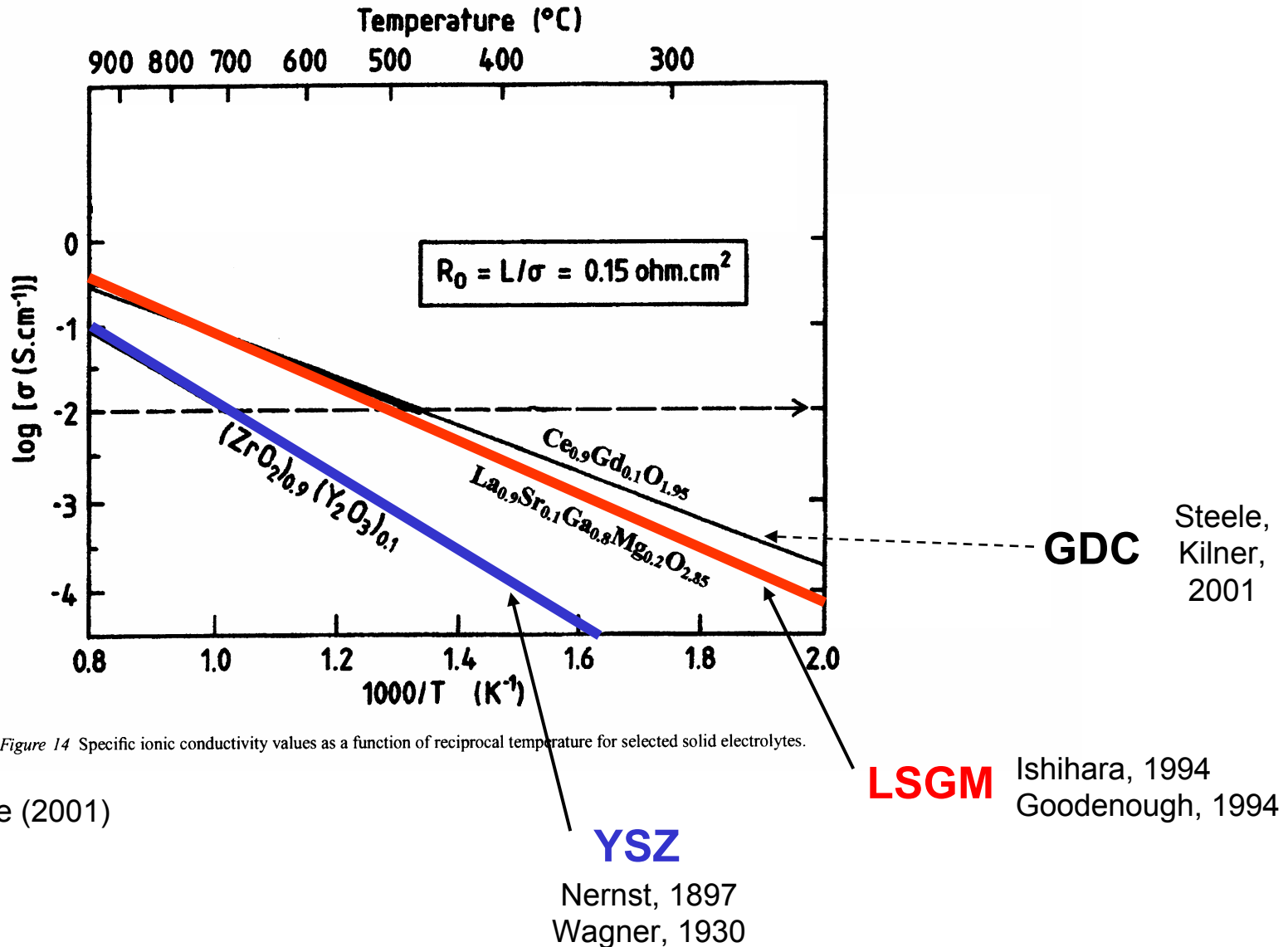


Figure 14 Specific ionic conductivity values as a function of reciprocal temperature for selected solid electrolytes.

B.C.H. Steele (2001)

## Cation diffusion in oxygen ion conductors

- Why study cation diffusion in an oxygen ion conductor?

- Cations are the slowest species:  $t_{\text{cation}} \ll t_e, t_h \ll t_o$   
 $10^{-10} \quad 10^{-4} \quad 1$



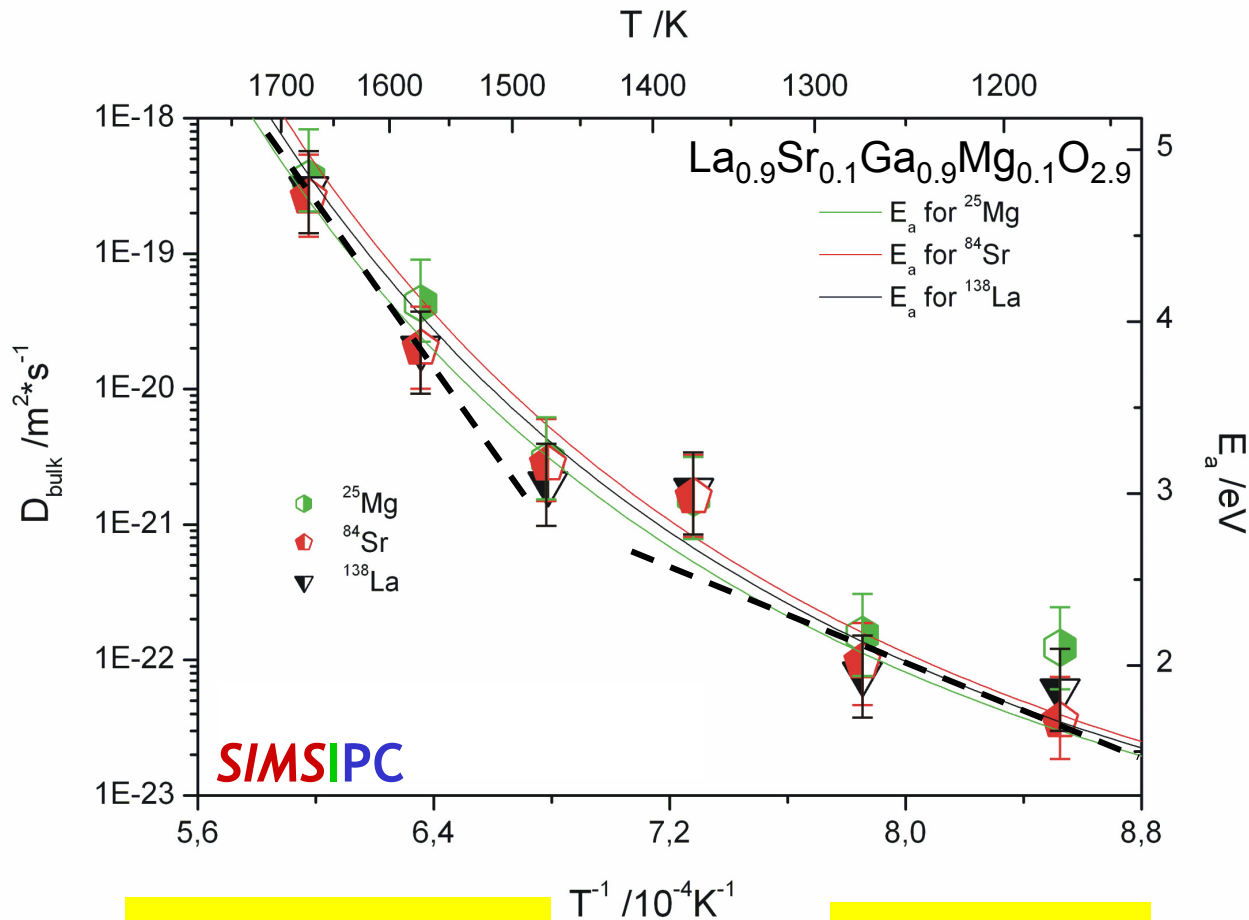
Minority  
defects

- They determine the rates of:
  - defect equilibration
  - sintering, creep
  - interdiffusion (with electrodes)
  - kinetic demixing (of electrolyte)

Long term  
degradation effects

- Complicated diffusion mechanism

# Cation self-diffusion in LSGM



- curvature

frozen in defects at low temperatures

defect formation  
+ migration  
 $\approx 4.5 \text{ eV}$

migration  
 $\approx 2 \text{ eV}$

# Cation self-diffusion in LSGM

## Experiment

- similar migration energies for all cations  
 $\approx 2$  eV
- similar diffusion coeff. for all cations

## Ionic radii

### A-site

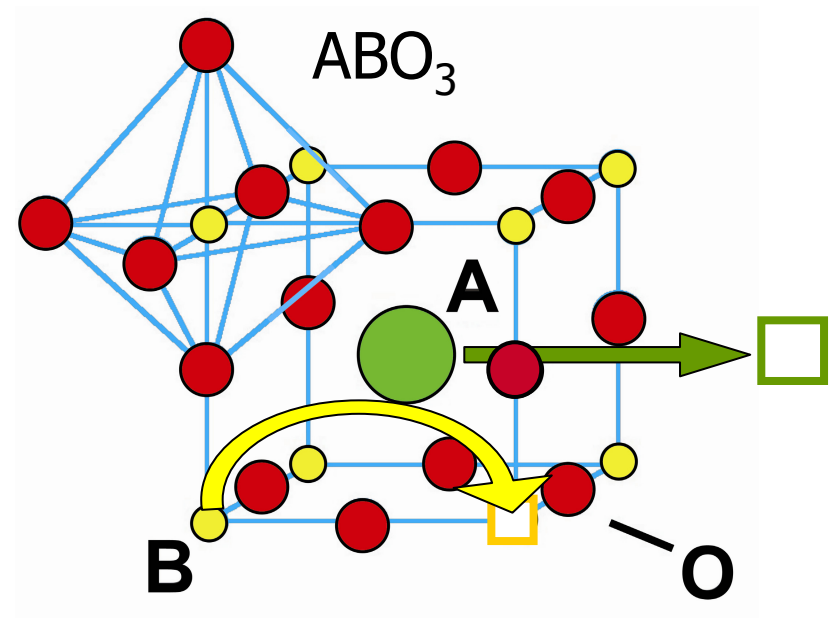
La<sup>3+</sup>: 106 pm

Sr<sup>2+</sup>: 118 pm

### B-site

Ga<sup>3+</sup>: 62 pm

Mg<sup>2+</sup>: 74 pm



## Theory

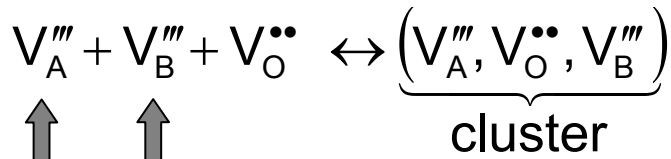
$$E_{\text{La}}^{\text{mig}} = 4.6 \text{ eV} \quad E_{\text{Ga}}^{\text{mig}} = 17 \text{ eV} \quad \text{Khan et al. (1998)}$$

$$E_{\text{Y}}^{\text{mig}} = 2.6 \text{ eV} \quad E_{\text{Cr}}^{\text{mig}} = 10 \text{ eV} \quad \text{De Souza, Maier (2002)}$$

More complicated  
diffusion mechanism ?

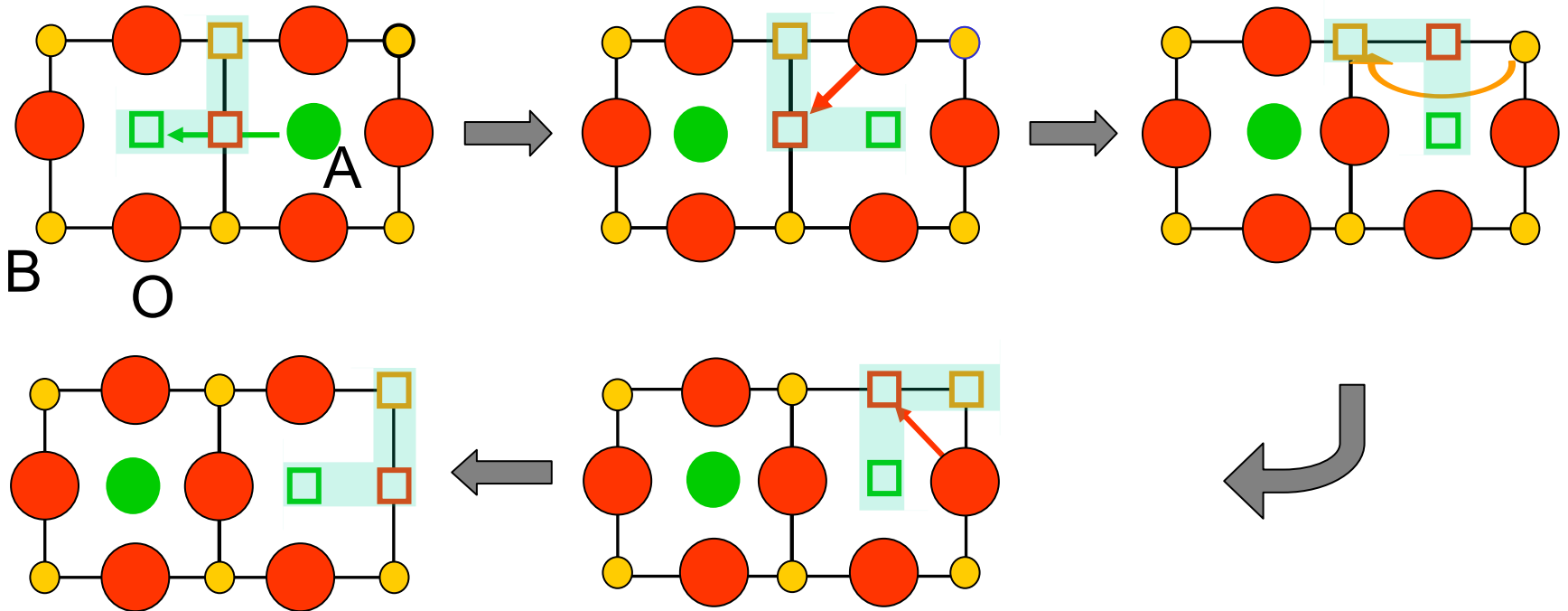
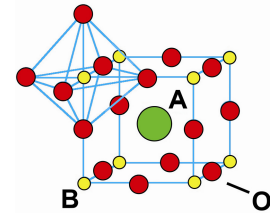
## Simple cation vacancy mechanisms

# Cluster formation in LSGM



$\Delta E_C \approx -7.5 \text{ eV}$   
(point charges)

Minority defects

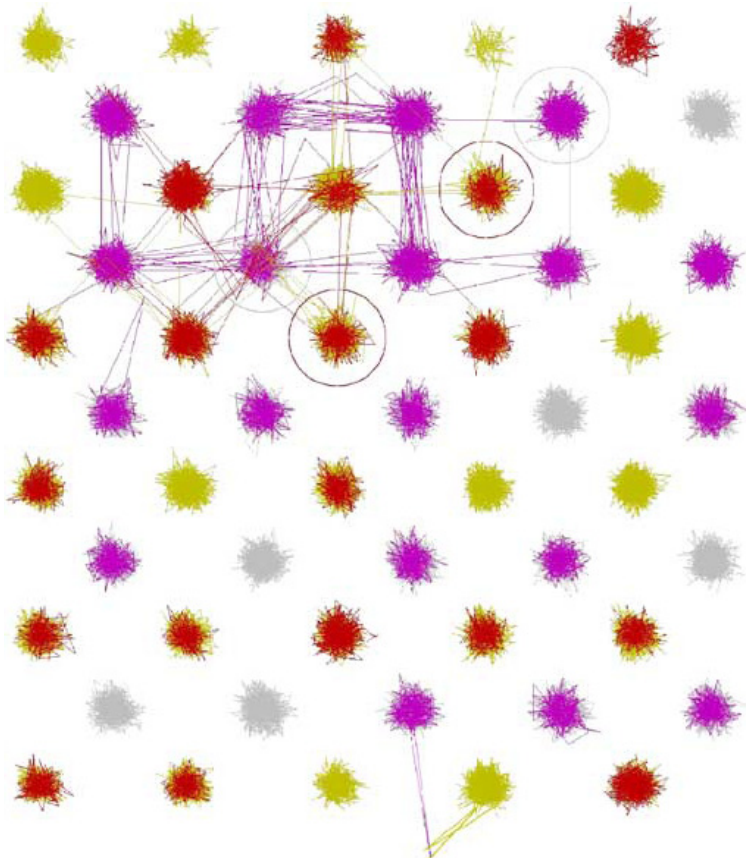


**Correlated four jump cycle**

$$\omega_O \gg \omega_A > \omega_B$$

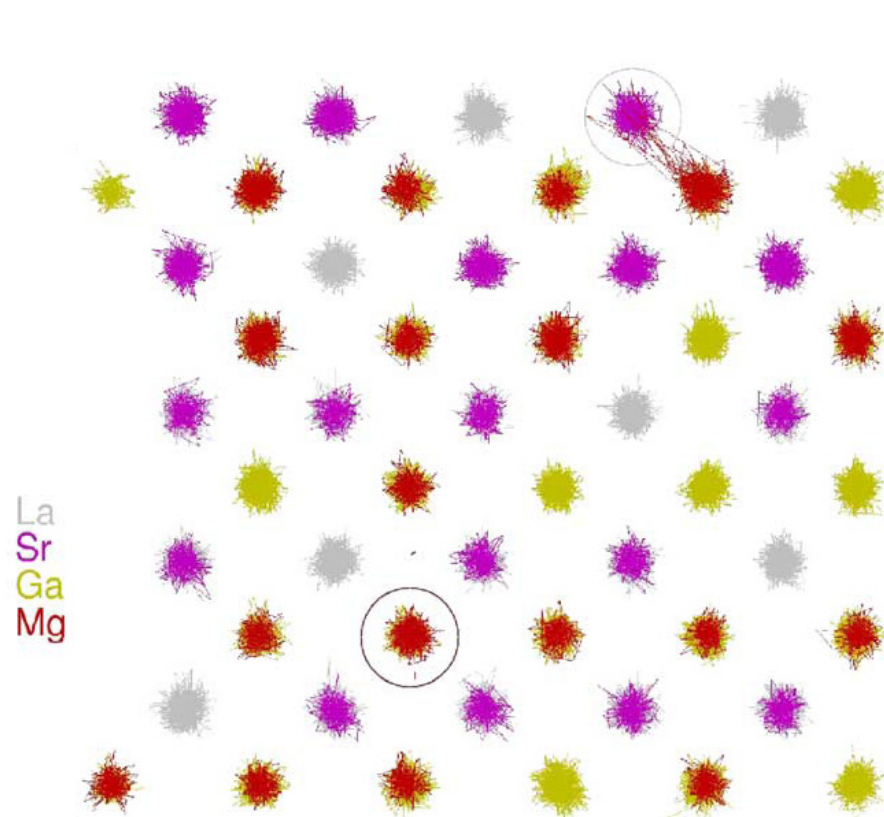
# Molecular Dynamics

Vacancy pair: ( $V_A, V_B$ )



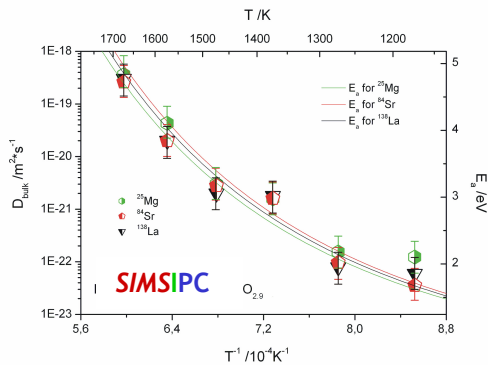
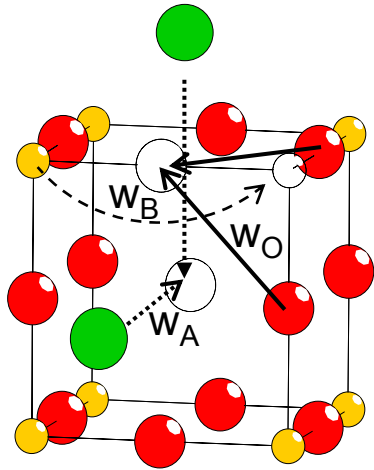
long range B-jumps via  $V_A$

Separated vacancies  $V_A$  and  $V_B$



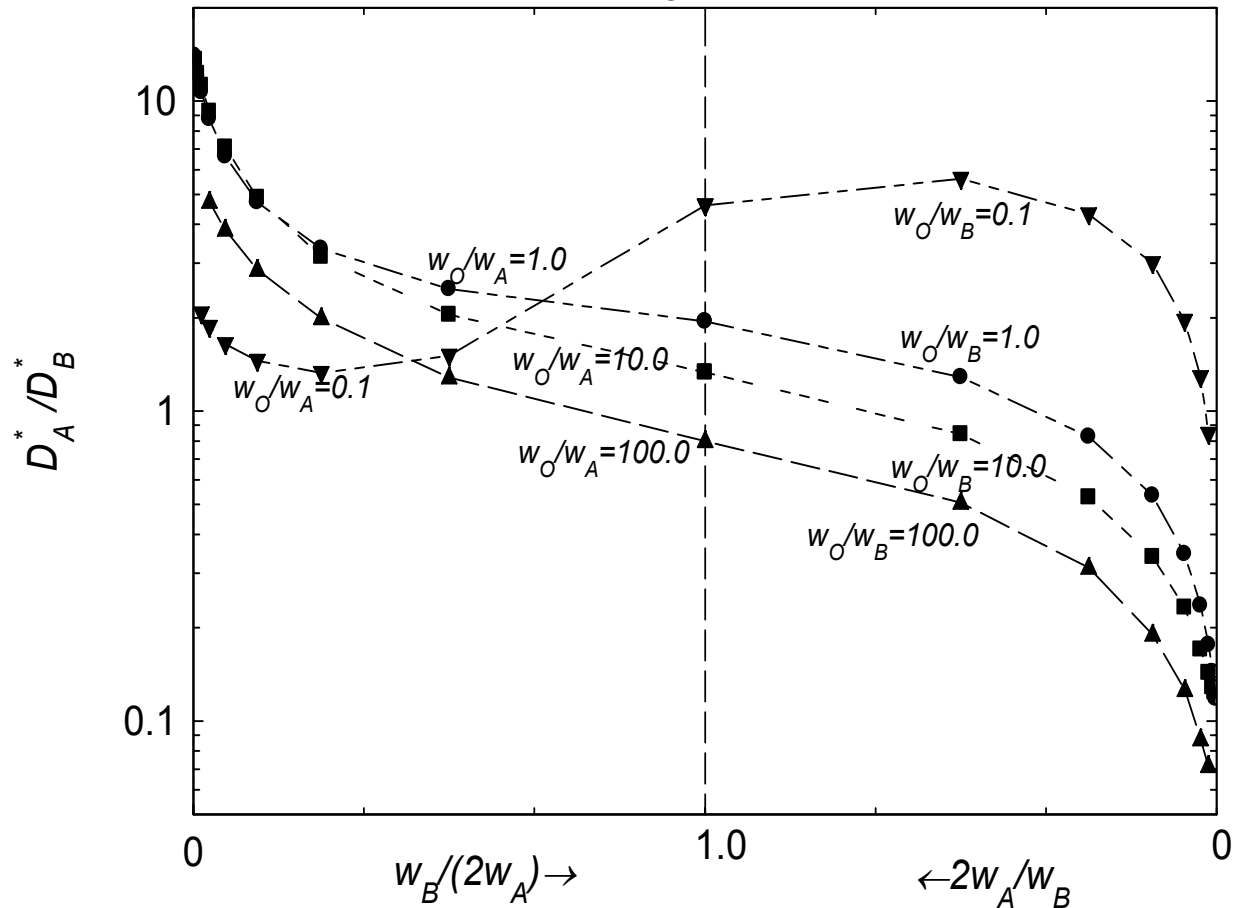
only forward-backwards jumps

# Cluster migration in $ABO_3$



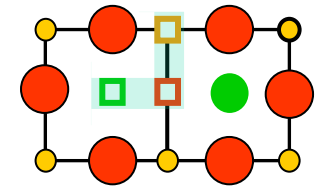
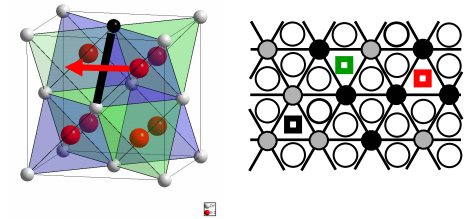
Sum rule expressions for the transport coefficients  $L_{ij}$ , e.g.

$$L_{AA} + 2 \frac{W_A}{W_O} L_{AO} + 2 \frac{W_A}{W_B} L_{AB} = L_{AA}^{(0)}$$



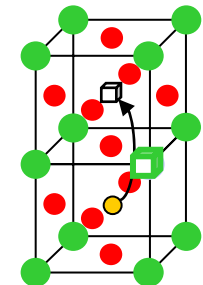
## Conclusions

- Oxygen diffusion in YSZ (and LSGM):
  - jumps through edges and interactions
  - simple, analytical model for concentrated defects and conductivity maximum
- Cation diffusion in LSGM (minority defects)
  - “identical” activation energies for all cations
  - “identical” diffusion coefficients for all cations
  - defect clustering
  - four-jump cycle
  - qualitative confirmation by GULP, MD and sum rules



- General mechanism for cation diffusion in perovskites ?

$$D_B \propto [V_B] \cdot [V_A]$$



- Thanks: DFG (SPP 1060), CHF, Land NRW



Vacancies,  
in Rome already known 27 BC

