Diffusion on random site percolation clusters. Theory and NMR microscopy experiments with model objects

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Quasi two-dimensional random site percolation model objects were fabricated based on computer generated templates. Samples consisting of two compartments, a reservoir of H\textsubscript{2}O gel attached to a percolation model object which was initially filled with D\textsubscript{2}O, were examined with NMR (nuclear magnetic resonance) microscopy for rendering proton spin density maps. The propagating proton/deuteron inter-diffusion profiles were recorded and evaluated with respect to anomalous diffusion parameters. The deviation of the concentration profiles from those expected for unobstructed diffusion directly reflects the anomaly of the propagator for diffusion on a percolation cluster. The fractal dimension of the random walk, \(d_w\), evaluated from the diffusion measurements on the one hand and the fractal dimension, \(d_f\), deduced from the spin density map of the percolation object on the other permits one to experimentally compare dynamical and static exponents. Approximate calculations of the propagator are given on the basis of the fractional diffusion equation. Furthermore, the ordinary diffusion equation was solved numerically for the corresponding initial and boundary conditions for comparison. The anomalous diffusion constant was evaluated and is compared to the Brownian case. Some ad hoc correction of the propagator is shown to pay tribute to the finiteness of the system. In this way, anomalous solutions of the fractional diffusion equation could experimentally be verified for the first time.