5 problems that a theory of lexical meaning should address

- types and selectional restrictions
- subtyping
- dual aspect terms or terms with multiple types.
- coercion
- shifts of lexical meaning in discourse contexts

Using a categorial setting is important to dealing with the 1st, 2nd and 3rd problems
Types in natural language semantics

- count vs. mass
- kind vs. individual
- abstract vs. concrete (informational object vs. physical object)
- eventualities vs. objects
- propositions vs. facts vs. eventualities
- locations vs. objects
- animate vs. non-animate

(1) John swept the kitchen
(2) John swept the dust
(3) #John swept the dust and the kitchen.

The subtyping problem

- a lot of linguistic evidence for a rich system of subtypes of E (eventualities vs. physical objects vs. informational or abstract objects vs. animate objects vs. locations vs. kinds vs. masses vs. ...)
- Most linguists working on lexical meaning assume these distinctions in more or less formalized form
- Lots of subtyping relations, entertained. E.g. physical objects are a subtype of objects. OK for Church/Montague set theoretic conception of types.
- But for higher order types, Church/Montague is a disaster. First order physical properties are intuitively a subtype of first order properties—\((P \rightarrow T) \subseteq (E \rightarrow T)\)
- Not so on the Church/Montague set theoretic model of types.
- In fact the type of first order physical properties and the type of first order properties are disjoint according to Church/Montague.
Adding subtypes of $e$

Let us suppose we have a set of subtypes of $e$ partially ordered by subtype $\subseteq$

• The translation for $a$ should combine with the translation of $boy$, but it doesn’t for the moment.
  
  – $a$: $\lambda PQ \exists x: e(Px \land Qx)$ of standard type:
  
  – $boy$: $\lambda v: \text{animate} \text{Boy}(v)$ of type $\text{animate} \Rightarrow T$

• allow for presupposition justification, whenever the argument is a subtype of the type requirement of the predicate: subtype coercion (Luo 1994). If a predicate environment requires something of type $\alpha$ and its argument is of type $\beta \subseteq \alpha$, then the type presupposition is considered justified.

The problem of extending $\subseteq$ to higher types

• $\text{animate} \subseteq e$

• But $\text{animate} \Rightarrow T \cap e \Rightarrow T = \bot$ in a Church set theoretic model of types.

• the option of partial functions: $\text{informational object} \cap \text{physical object} \neq \bot$ or $\bot$ has inhabitants.
The solution in computer science

- the Curry-Howard isomorphism: types as proofs, intuitionist semantics for types
- what does this mean for linguistics? Proofs for mathematics replaced by defeasible derivations,
- types as highly intensional objects, concepts. Various criteria for concept equivalence (input-output, trace equivalence, bisimulation...)

**Fact 1**  Subtyping for functional types:

\[
\alpha \subseteq \alpha' \land \beta \subseteq \beta' \implies (\alpha \cup \alpha' \Rightarrow \beta \cap \beta') \subseteq (\alpha \Rightarrow \beta)
\]

This doesn’t seem quite right either.

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The categorial model

The semantics for types as abstract objects. It will also help us model the structure of the inhabitants of complex types.

categories: characterizing objects (types) by their relations to other objects, by the morphisms they allow (deductions).

we can go beyond the set-theoretic model of types to consider other constructions.
Some basic definitions

Definition 1  A graph consists of a set of objects (points) and a set of arrows between points (edges). In addition, there are two maps from Arrows to Objects, one labeled *Domain* and the other *Co-Domain*, which pick out the collections of first arguments and second arguments of arrows respectively.

Definition 2  A deductive system $D$ is a graph in which for each object $A$ in $D$, there is an arrow $1_A$, which is the identity map on $A$, and for each pair of arrows $f : A \rightarrow B$ and $g : B \rightarrow C$, $g \circ f : A \rightarrow C$, the composition of $g$ with $f$, is defined. Objects can be identified with formulas and arrows with proofs. The closure under composition gives us a rule of inference.

Categories

Definition 3  A category is a deductive system where $\forall f : A \rightarrow B$, $g : B \rightarrow C$ and $h : C \rightarrow D$:

- $f \circ 1_A = f = 1_B \circ f$
- $(h \circ g) \circ f = h \circ (g \circ f)$
In category theory one can define certain constructions over objects in a category. Products, for instance, are one such construction.

**Definition 4**

Let $\mathcal{A}$ be a category and $a, b$ be objects in $\mathcal{A}$. Then $a \times b$ is the categorial product of $a$ and $b$, which comes with two arrows $\pi_1: a \times b \to a$ and $\pi_2: a \times b \to b$ such that for any arrows $f: c \to a$ and $g: c \to b$ in $\mathcal{A}$ there is a unique $h: c \to a \times b$ such that $h \circ \pi_2 = g$ and $h \circ \pi_1 = f$.

Category theorists typically display a categorial product using the following diagram where all the arrows commute:

\[
\begin{array}{c}
\text{c} \\
\text{a} \\
\text{b}
\end{array}
\begin{array}{c}
\pi_1 \\
\pi_2
\end{array}
\begin{array}{c}
\text{f} \\
\text{g}
\end{array}
\]

In the category of Set, product constructions correspond to Cartesian products.

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**Initial and terminal objects**

Other constructions of interest in category theory are those of an *initial* and *terminal object*.

**Definition 5** An object $0$ is an initial object in a category $\mathcal{C}$ iff for all objects $c$ in $\mathcal{C}$, there is a unique arrow $f: 0 \to c$ in $\mathcal{C}$. An object $1$ is a terminal object in $\mathcal{C}$ iff for all objects $c$ in $\mathcal{C}$, there is a unique arrow $f: c \to 1$ in $\mathcal{C}$.

Terminal objects are the duals of initials.

Duality in category theory reverses the directions of the arrows in a construction. Categorial *coproducts* are the dual of products and correspond to disjoint union in set theory. They give rise to the following commuting diagram:

\[
\begin{array}{c}
\text{c} \\
\text{a} \\
\text{b}
\end{array}
\begin{array}{c}
\pi_1 \\
\pi_2
\end{array}
\begin{array}{c}
\text{f} \\
\text{g}
\end{array}
\]

In the category of Set, product constructions correspond to Cartesian products.
Cartesian closed categories

A cartesian category is one that contains a terminal object and that is closed under the product construction. Duality ensures that a cartesian category also contains coproducts.

A topos is a CCC with a subobject classifier (a way of characterizing subsets of sets in the category of SET).

Montague Grammar in category theory

A good way to understand the power of category theory is to link it to a familiar formal semantics, MG.

Montague’s categorial grammar is an example of a biclosed monoidal category. A monoidal category with an object 1 and an associative operation \( \otimes: \mathcal{C} \times \mathcal{C} \to \mathcal{C} \), in which objects include not only basic objects but also morphisms. Thus if \( a \) and \( b \) are objects, \( a \to b \) is also an object.

A closed monoidal category \( \mathcal{C} \) has a right adjoint \( a^{-1} \) for each object \( a \) in \( \mathcal{C} \) (takes \( a \) into 1 but can also take \( a \) into another object \( b \)).

\( N \to NP \) is then an adjoint to \( N \) that yields an \( NP \) in the presence of an \( N \); it is isomorphic to \( N^{-1} \otimes NP \).

An expression of syntactic type \( A \) is a morphism \( 1 \to A \).

Thinking of \( A \) as a set we can identify 1 unique morphism from \( 1 \to A \) for each element of \( A \). But this is only one way of thinking of types!
From syntax to semantics

- Every BMC can be embedded in a CCC and every CCC can be embedded in a topos.
- More complex categories simply add more structure, just as groups can be embedded within rings, or arithmetic within analysis.
- Thus each syntactic category is mapped into a semantic type via this morphism $f : \mathcal{C}_{bmc} \rightarrow \mathcal{C}_\tau$.
- An object of syntactic type $A$, i.e. a morphism $s : 1 \rightarrow A$ is mapped via $f$ to a semantic value of type $B, \sigma : 1 \rightarrow B$.
- The translation from syntactic derivations to semantic values takes place via this embedding.
• A Cartesian Closed Category furnishes the basics, providing a model of $\Rightarrow$, $\lor$ and functional and disjunctive types.

• a topos is a CCC plus characteristic functions for subsets (subobject classifiers)—think of a subset $a$ of $b$ as an embedding $m: a \rightarrow b$.

• Consider the ordinary way of thinking of a semantic value of a 1 place predicate as a function from individuals into $\{\top, \bot\}$ (the characteristic function of the set denoted by the predicate). In a topos $\{\top, \bot\}$ is the distinguished object $\Omega$, with the structure of a Boolean algebra. We can mimic this if we wish...

• Lambek's proposal: identify sentence meanings with $1 \rightarrow \Omega$. This yields only a purely extensional theory. Only two propositions!

• types are concepts not identified with sets of their inhabitants but rather something like proof objects with rules of application

• So we distinguish a Pre Boolean Algebra object $\text{PROP}$ distinct from $\Omega$. $\text{PROP}/\equiv$ is a BA, where $\equiv$ is the equivalence relation induced on $\text{PROP}$ by the usual Boolean identities. (Pollard 2011)

• $\text{PROP}$ is inductively constructed from a basic set of subtypes of $E$, not to be confused with their extensions. Boolean operations and standard quantifiers can be defined within the topos.

• $\Omega$ plays the counterpart of Montague's type $T$. 

Rethinking semantic values within toposes
Fine grained intensionality and proof objects

- Category theory is ideal for characterizing types/concepts, because it allows us to characterize them without details of internal structure.
- Can use \( \equiv \) to provide BA-like equivalences over properties as well as propositions without collapsing concepts that are equivalent.
- Think of them as proof theoretic objects: a simple type \( A \) gives us instructions for finding morphisms \( 1 \to A \):
  - \( \alpha \sqsubseteq \beta \): a proof that something is \( \alpha \) furnishes a proof that that thing is also \( \beta \)
  - \( \alpha \to \beta \) if one has a proof that \( x \) is \( \alpha \), then that can be extended to a proof that it is \( \beta \).
- First order properties defined as \( \Sigma x \subseteq E (x \Rightarrow T) \), and physical first order properties are defined as \( \Sigma x \subseteq P (x \Rightarrow T) \) where \( \Sigma \) is an existential quantifier over types.
- Given that \( P \sqsubseteq E \) physical properties are first order properties.

Subtyping revisited

- The categorial structure of topoi gives us an inference relation but it is too unconstrained.
- Need to rule out for instance the fact that any \( \alpha \to \alpha \) can be inferred from anything.
- Restrict the set of \( \to \)'s relevant to subtyping to those just generated over a given type hierarchy for the simple subtypes of \( E \) encoded in \( \Delta \).
  - If \( \alpha \sqsubseteq_{ST} \beta \), then \( \alpha \to \beta \in \Delta \)
  - \( \Delta, \alpha \vdash \beta \) and \( \Delta \not\vdash \beta \)

**Definition 6** Subtyping: \( \alpha \sqsubseteq \beta \) iff \( \alpha \vdash_{\Delta} \beta \)
Dual aspect nouns (4th problem)

(4) Ducks lay eggs and are common to most of Europe. [individual and kind]

(5) Snow is common this time of year and all over my back yard. [kind and mass]

(6) Mary picked up and mastered three books on mathematics. [physical object and informational content]

(7) Je ne suis qu’un roseau mais je suis un roseau pensant. (Pascal) [physical object and thinking agent]

(8) That is a lump of bronze but also Bernini’s most famous famous statue [portion of matter and artifact]

(9) Le prix Goncourt is 10000 euros and a great honor not accorded every year. [amount of money and prize]

(10) The lecture (interview, speech) lasted an hour and was very interesting. (event and information)

(11) The bank is just around the corner and specializes in sub prime loans. (physical object/location and institution).

(12) The canvas is immense, and, as an example of a bygone school, interesting to all art lovers. (New York Times March 24, 1894) [physical object and informational object]

(13) The promise was made but impossible to keep. [speech act (event) and proposition]

(14) The belief that all non Christians are immoral is false but persists in many parts of America. [informational content and state]

(15) Mary’s testimony concerning the matter occurred yesterday and is riddled with factual errors and inconsistencies [event and information content]

(16) The very same thought that caused Mary to shudder amused Kim. [event and information content]

(17) The house contains some lovely furniture and is just around the corner. [physical object and location]

(18) Most cities that vote democratic passed anti-smoking legislation last year. [population and legislative entity]

(19) Lunch was delicious but took forever. [food and event]

(20) The temperature is 90 and rising. function and value
Copredication, counting and quantificational shift with dual aspect nouns

- dual aspect nouns (book, lunch, city)—shifts in quantificational domains (quantificational puzzles)
- mass nouns, bare plurals (at the level of determiners)

(21) John has read every math book in the library.
(22) John has mastered every math book in the library.
(23) John has stolen every math book in the library
(24) John answered every question.
(25) John repeated every question.

This is important because we see that verbs can select aspects of nouns that force shifts in the domain. So typing here isn’t just a matter of provability.

Analyzing dual aspect nouns and copredication

- a simple ambiguity (lots of ambiguities don’t support copredication)

(26) The bank is slippery from the recent rains and specializes in IPOs.

- copredications are ellipses; copy the DP with the dual aspect noun into the second predication and disambiguate. (predicts false readings for quantified sentences like Mary carried home and then mastered 3 books on semantics)

- dual aspect nouns have a conjunctive type: BOOK ⊑ I ⊓ P.
  (but no object is both an individual instance of a kind and a kind...)

- dual aspect nouns have a product type:
  BOOK ⊑ I ⊔ P
  Better, but...

Better, but...
Counting arguments

a there are exactly one copy combining War and Peace and Anna Karenina two copies of Ulysses, and six copies of the Bible on a shelf.
b Pat has read War and Peace and Ulysses, and no other book
c Sandy has read the Bible, and no other book.

Now, consider the questions:

• (Q1) How many books are there on the shelf?
• (Q2) How many books has Pat read?
• (Q3) How many books has Sandy read?
• (Q4) Who has read more books, Pat or Sandy?

My guess is that most people would answer:

• 9 to (Q1)
• 2 to (Q2)
• 1 to (Q3)
• Pat to (Q4)

Pairing approach gives us too many objects

The pairing approach gives us 10 pair objects consisting of a physical object represented by a number of the physical copy and an information content represented by the title of the book or an abbreviation thereof:

• (1, W&P), (1, AK), (2, U), (3, U), (4, Bible), (5, Bible), (6, Bible), (7, Bible), (7, Bible), (8, Bible), (9, Bible).

Also a problem with anaphoric dependencies. And we don’t want to shift the lexical type of book!

(27) The book is not easy to carry around, as it weighs 5 lbs. But it is the best and most interesting treatment of topoi.

Need a local variable that’s of type P and one of type I that are linked to book

On the other hand verbal predications can shift the domain of quantifiers.
Why Complex Types are complex

Consider a model where • types are modelled as conjunctive types.

• Conjunctive Types Axiom:
  \[ x : \alpha \cap \beta \text{ iff } x : \alpha \land x : \beta \]

This model of • adopts the following conjunctive types hypothesis (CTH). Let’s assume for simplicity, as most work in lexical semantics does, that the type hierarchy forms a complete lattice in which greatest lower bounds are assured to exist. Then:

• Conjunctive Types Hypothesis (CTH):
  \[ \alpha \bullet \beta := \alpha \cap \beta = \text{glb}\{\alpha, \beta\} \]

CTH’s Account of Copredication

(28) The book is interesting but very heavy to lug around.

• Imagine that the coordinated adjectival phrases place a • type requirement on its argument—e.g., P • I.
• Types match; copredication succeeds.
Problem with CTH

For many cases of copredication, not the case that for $x : \alpha \bullet \beta$ that $x : \alpha$ and $x : \beta$, because $\alpha \cap \beta = \bot$. By CTH, $\alpha \bullet \beta = \text{glb}\{\alpha, \beta\} = \bot$.

- complex speech acts of the form QUESTION • REQUEST, QUESTION $\cap$ REQUEST = $\bot$.
- LUNCH $\subseteq$ EVENT • FOOD. EVENT $\cap$ FOOD = $\bot$. Events and physical objects have different, even incompatible properties. Objects, from our commonsense point of view, perdure through time while events have a duration; objects are wholly present at each moment in time while events have temporal parts. Linguistic tests: temporal modifiers

(29)  a. The tree grew slowly.
    b. The tree was slow.
    c. The tree was slow in growing.

- AUSTIN $\subseteq$ CITY $\subseteq$ LOCATION • ADMINISTRATIVE BODY. But ADMINISTRATIVE BODY has 8 members. Locations don’t have members (perhaps inhabitants, of which Austin has about 1 million.

Physical and Informational Objects

- BOOK $\subseteq$ PHYSICAL-OBJECT • INFORMATIONAL-OBJECT. Informational objects and physical objects also have incompatible individuation properties. Informational objects, abstract objects that contain information can have multiple concrete instantiations, if indeed they have any physical instantiations at all. Individual physical objects cannot have multiple concrete instantiations.
A Case Study about Books (Tim Fernando)

a there are exactly two copies of War and Peace,
two copies of Ulysses, and six copies of the Bible
on the shelf
b Pat has read War and Peace and Ulysses, and no
other book
c Sandy has read the Bible, and no other book.

Now, consider

• (Q1) How many books are there on the shelf?
• (Q2) How many books has Pat read?
• (Q3) How many books has Sandy read?
• (Q4) Who has read more books, Pat or Sandy?

TF’s guess is that most people would answer:

• ten to (Q1)
• two to (Q2)
• one to (Q3)
• Pat to (Q4)

Problem Continued

if \( glb\{\alpha, \beta\} = \bot \) in virtue of incompatible properties
of their inhabitants, \( \text{Inh}(\alpha \bullet \beta) = 0 \)

Since there are obviously books, then if we assume
that books have a dual nature in some sense yet to
be specified, CTH must be wrong.
(30)  a. The apple is red.
    b. The apple is juicy (is delicious).

\[ x : \text{APPLE} \bullet \text{SKIN} \rightarrow, \text{according to CTH, } x \text{ is the skin of the apple. So the intersection of the type SKIN and FOOD just give us the skin of the food, and that’s not what tastes delicious or is juicy.} \]

Another Problem

Another Hypothesis

Using Category Theory as an abstract model for types, we can say that \( \alpha \bullet \beta := (\alpha, \beta) \), the product construction.

- **Pair Types Hypothesis (PTH):**
  \( \alpha \bullet \beta := (\alpha, \beta) \)

- Makes sense of different individuation conditions, say of books. Use one element or the other of the pair to give the appropriate counting principle (Gech, Gupta, Baker).

- Once again makes sense of copredication (same story as for intersective types, but replace products for the type restriction of the coordinated predicates)
Simple Predications with PTH

(31) The book weighs five pounds

• \( \lambda x \) weighs five pounds \( (x) \rightarrow x : \mathbb{P} \)

• Assume projection operations \( \pi_1, \pi_2 \) on types such that \( \pi_i(\tau) = i \)th constituent of \( \tau \), if \( \tau \) is a, ow. undefined.

• ?? \( \lambda P(\exists x(\text{book}(x) \land P(x))), \ x : \mathbb{P} \rightarrow x : \pi_1(\mathbb{P} \bullet 1) \)

(28b) John’s Mom burned the book on magic before he could master it.

If we shift the book on magic to \( \mathbb{P} \) so as to make the predication in the main clause of (28b) succeed, then the anaphoric pronoun will not have an antecedent of the appropriate type for the predication in the subordinate clause.

PTH Modified

The crucial insight seems to be that the projections from complex types go with different terms not the original term.

Assume to function symbols \( f_1 \) and \( f_2 \) that give us new terms associated with \( t \).

• Separate Terms Axiom (STA):

\[ t : \alpha \bullet \beta \text{ iff } f_1(t) : \alpha \wedge f_2(t) : \beta \]

With the relevant details now omitted, the analysis of

(32) The book weighs five pounds and is an interesting story.

under PTH and STA would have the following logical form:

\[
\exists x(\text{book}(x) \land \text{weighs five pounds}(f_1(x)) \land \text{interesting story}(f_2(x)),
\langle x : \{\text{INFO-OBJ, PHYS-OBJ}\}, f_1(x) : \pi_1(\{(\text{PHYS-OBJ, INFO-OBJ})\}),
f_2(x) : \pi_2(\{(\text{PHYS-OBJ, INFO-OBJ})\})\rangle)
\]
An incompleteness in PTH+STA

- Not always a functional relation between elements of a constituent type and elements of the complex type (books as $p \cdot i$ might be individuated via their information conditions and so have many physical instantiations).
- Some dependence between $f_i(t)$ and $t$ where $t : \alpha_1 \cdot \alpha_2$ and $f_i(t) = \alpha_i$. But what is it?
- What does $t : (\alpha, \beta)$ tell us about semantic values of $t$?

A Mereological Hypothesis (Cooper)

- Inhabitants of $\bullet$ types are in fact collections or sums of parts of simple types.
- Lunch has an event part and a food part. Is lunch a singular noun referring to a plural collection of its parts?

(33) a. The orchestra got ready. Then they started to play the Bach suite.
    b. The battalion was in trouble. They were receiving heavy fire from the enemy. They called in for tactical air support.
    c. I thought the lunch for Chris Peacock was very nice. They pleased him too.

- Inhabitants of $\bullet$ types are not collections! Lunch is one thing not two!
A Metonymic Conception (Kleiber and Cooper)

• Inhabitants of types have parts picked out by the simple constituent types.

• So, e.g., a lunch is a single item but composite a fusion, of an event and foodstuff. lunches are like apples with several parts.

• the parts of apples are named, but not so for lunches.

• The parthood relation holding between the parts of an apple and the apple as a whole is not the same as the parthood relation operative in lunches or books. Not a physical parthood relation for books.

• Allowed by unrestricted mereological composition. But is that a plausible theory of object-hood? Not substance dualism but substance "multi-ism"

• A Benaceraff type problem:
  (34) Part of the lunch is an event and part of the lunch is a meal.

Aspects and Tropes

• Leave the objects simple, but complicate the notion of predication.

• Relative predication as a guide:
  (35) John as a janitor makes only $20K but as a salesman on E Bay he makes $40K.
  (36) That statue as a lump of stone is undistinguished but as a statue by Bernini it is one of the greatest works of art.
  (37) This book as a paddle is useless.
  (38) a. An isosceles triangle as a triangle has two equal sides.
      b. An isosceles triangle as such (as an isosceles triangle) has two equal sides.

• Predication relative to a conceptualization of the object (an aspect or guise). Kratzer’s and others’ notion of a "thick object"

• Aspects are not constitutive of objects (not parts in the normal sense). They form a partial order, however, under entailment with the aspect True(a) the sup of all aspects of object a (Asher 2006).
Upshot: dual aspect nouns require a special type

- \textbf{BOOK} \sqsubseteq \textbf{I} \bullet \textbf{P}
- model this type as sui generis but licensing a natural transformation to a fibre product, which says, roughly, that given a maps from one aspect to another, there exists an object that has these two aspects.
- the categorial model is well adapted to conceiving of types as proof objects
- copredication involves a type justification in which an aspect type is selected, and this may affect the domain.
- the selection of an aspect type will determine counting and individuation principles and will shift domains of the quantifiers accounting for the quantificational puzzle.

Some counting pictures

On the shelf is a copy of Jane Austen’s collected works and 3 copies of \textit{The Bible}:

![Figure 1: Books individuated physically](image1)

![Figure 2: Books individuated informationally](image2)
I distinguish a particular class of transformations, I’ll call them *Aspect* or *A*, from one topos $C$ that has $\bullet$ types to another $D$ containing a new arrow from the bullet type to the pullback of the right sort.

**Definition 7** $\alpha \times \beta[R]$ is the restriction of the product/pullback $\alpha \times \beta$ such that for the projections $\pi_1$ and $\pi_2$ of $\alpha \times \beta \pi_1(x) = y$ and $\pi_2(x) = z$ iff $R(y, z)$

We can now graph the natural transformation $A$ and its associated arrow, *aspect*, as in (3). Assume that the natural relationship constitutive of $\alpha \bullet \beta$ is $R$.

![Diagram of natural transformation and associated arrow](image)

**Pullbacks**

The “I” pullback

![Diagram of “I” pullback](image)

The “P” pullback

![Diagram of “P” pullback](image)
Consequences of the model

I postulate domains for each type. Domains $D_\alpha$ for simple types $\alpha$ have their own individuation and counting criteria provided by $\alpha$. But what about the inhabitants of $\bullet$ types? Could they simply be identical to those that inhabit the simpler types?

Lemma:

- $t_1 = t_2 \rightarrow (t_1 : \alpha \leftrightarrow t_2 : \alpha)$

or, given that typing contexts are functions from terms to types:

- (ITID) $t_1 = t_2(\pi) \rightarrow \text{TYPE}_\pi(t_1) = \text{TYPE}_\pi(t_2)$,
  where $\pi$ is the type parameter for the formula $t_1 = t_2$.

ITID follows from TCL as a theorem, if we assume the following lexical entry for *is identical to*:

- *is identical to: $\lambda x \lambda y \lambda \pi x = y(\pi \ast \text{TYPE}(x) = \text{TYPE}(y))$

However, in general, $\alpha \bullet \beta \neq \alpha$. In particular, if $\alpha \cap \beta = \bot$, then $\alpha \bullet \beta \neq \alpha$, because objects of $\bullet$ type have some properties incompatible with the properties of the objects of type $\alpha$. Together with the principle of identical types for identicals, we now see if $x : \alpha \bullet \beta$ and $y : \alpha$ in any typing context $C$, then we can immediately infer $x \neq y$. Now suppose that $D_\alpha \bullet \beta$ and $D_\alpha$ have a common inhabitant. This should make (39) true:

(39) $\exists x : \alpha \exists y : \alpha \bullet \beta x = y$

We now derive an immediate contradiction from ITID.

**Fact 2** the inhabitants of $\alpha \bullet \beta$ must be disjoint from the inhabitants of $\alpha$ and $\beta$, unless $\alpha = \beta$
Counting arguments again

Consider the Jane Austen case again, where we have one physical volume containing Jane Austen’s seven published novels together with three copies of the Bible on a shelf.

Whereas if we individuate with respect to the \( i \) type, we have the picture in (5).

We assume there are simple types and five sorts of complex types, \( \bullet \)-type, disjunctive types and dependent types, as well as their associated functional types (e.g. \( \alpha \Rightarrow \beta \)) corresponding to \( \lambda \)-expressions:

(40) a. **PRIMITIVE TYPES**: a set including \( e \) the general type of entities and \( T \) the type of truth values forming a complete lattice under \( \sqsubseteq \).

b. **FUNCTIONAL TYPES**: If \( \sigma \) and \( \tau \) are types, then so is \( (\sigma \Rightarrow \tau) \)

c. **DOT TYPES**: If \( \sigma \) and \( \tau \) are types, then so is \( (\sigma \bullet \tau) \)

d. **DISJUNCTIVE TYPES**: If \( \sigma \) and \( \tau \) are types, then so is \( (\sigma \lor \tau) \) (to handle cases of homonymous ambiguity).
From categorial models to type presupposition justification

The categorial model of \( \bullet \) gives a relatively well-behaved notion of subtype for \( \bullet \).

But in general, \( \bullet \) subtype hierarchies are distinct from ST or the hierarchies involving their type constituents.

I now turn to presupposition justification

An example

(41) a heavy book

The interpretation of interest is the predication of the property of being heavy to the book *qua* physical object.

(42) \( \lambda P \lambda x: E \lambda \pi P(\pi^\ast\text{ARG}^\text{book}_1. P \bullet 1)(x)(\lambda v \lambda \pi' \text{book}(v, \pi')) \)

where \( P: \text{MOD, or } 1 \Rightarrow 1 \). This combines via Application with *heavy*'s entry, which is given in (43):

(43) \( \lambda P: 1 \lambda u \lambda \pi_1 (\text{heavy}(u, \pi_1^\ast\text{ARG}^\text{heavy}_1 . P) \land P(\pi_1)(u)) \)

(42) and (43) combine to give us:

(44) \( \lambda x \lambda \pi \lambda P \lambda u \lambda \pi_1 (\text{heavy}(u, \pi_1^\ast\text{ARG}^\text{heavy}_1 . P) \land P(\pi_1)(u)) \)

\[ [\pi^\ast\text{ARG}^\text{book}_1. P \bullet 1](x)(\lambda v \lambda \pi' \text{book}(v, \pi')) \]
Continuing the derivation

A use of Application and Substitution gives us:

\[ \lambda x \lambda \pi \lambda P \lambda u \left( \text{heavy}(u, \pi \ast \text{ARG}_1^{\text{book}}: P \bullet I \ast \text{ARG}_1^{\text{heavy}}: P) \land P(\pi \ast \text{ARG}_1^{\text{book}}: P \bullet I)(u)(x)(\lambda v \lambda \pi' \text{book}(u, \pi')) \right) \]

\text{ARG}_1^{\text{book}}: P \bullet I \text{ is, recall, an instruction that this type must be satisfied by the types assigned to the variables going in for that argument of book in the lambda term to which it is an argument. The problem is that } u \text{ is both the first argument to heavy and the first argument of book. So it has to obey incompatible typing requirements.}

Taking aspects seriously

\textit{heavy} should be predicated of the physical aspect of \textit{book}.

But \textit{heavy} shouldn’t shift the type of \textit{book}, it must pick out some aspect.

So we need a variable of complex type that will merge with the \( \lambda \) bound variable representing the book \textit{and} a variable of type \( P \) to combine with \textit{heavy}.

We link the two variables via a o-elab (object elaboration) relation.

Combining this requirement together with all our other constraints, we must justify the \( \bullet \) type presupposition of \textit{book} by adding material to the adjective.
Implementation

We add this material by applying a functor to the term that can’t justify the presupposition so as to turn it into one that can.

The term that causes the problem is:

\[ \text{heavy}(u, \pi_1 \text{ARG}^\text{book}_1: \text{P} \bullet \text{I} * \text{ARG}^\text{heavy}_1: \text{P}). \]

Via Abstraction this is equivalent to:

\[ \lambda v: \text{P} \lambda \pi_1 \text{heavy}(v, \pi_1)(\text{ARG}^\text{book}_1: \text{P} \bullet \text{I} * \text{ARG}^\text{heavy}_1: \text{P})(u). \]

We will apply a functor of the form (46) to the abstracted term which is a physical first order property to turn it into something that will justify the presupposition.

(46) \[ \lambda P \lambda w: \text{P} \bullet \text{I} \lambda \pi_3 \exists z: \text{P} (P(\pi_3)(z) \land o\text{-elab}(z, w, \pi_3)) \]

The whole term in (46) has the type of mapping a physical property into a property of \( \bullet \) type objects.

Returning to the derivation...

Let's now return to (45) and the derivation of a heavy book. (45) is equivalent via Abstraction to

(47) \[ \lambda x \lambda \pi \lambda P \lambda u (\lambda v \lambda \pi_1 \text{heavy}(v, \pi_1)(\text{ARG}^\text{book}_1: \text{P} \bullet \text{I} * \text{ARG}^\text{heavy}_1: \text{P})(u) \land P(\pi * \text{ARG}^\text{book}_1: \text{P} \bullet \text{I})(u))(x)(\lambda v \lambda \pi' \text{book}(v, \pi')) \]

The application of the functor in (46) to the term, \( \lambda v \lambda \pi_1 \text{heavy}(u, \pi_1) \), yields:

(48) \[ \lambda P \lambda w \lambda \pi_3 \exists z (P(\pi_3)(z) \land o\text{-elab}(z, w, \pi_3))[\lambda v \lambda \pi_1 \text{heavy}(v, \pi_1)] \]

A use of Application yields:

(49) a. \[ \lambda w: \lambda \pi_3 \exists z (\lambda v \lambda \pi_1 \text{heavy}(v, \pi_1)[\pi_3](z) \land o\text{-elab}(z, w, \pi_3)) \]

b. \[ \rightarrow_{\beta} \lambda w: \lambda \pi_3 \exists z (\lambda v \text{heavy}(v, \pi_3)[z] \land o\text{-elab}(z, w, \pi_3)) \]

c. \[ \rightarrow_{\beta} \lambda w: \lambda \pi_3 \exists z (\text{heavy}(z, \pi_3) \land o\text{-elab}(z, w, \pi_3)) \]

Let us now reintegrate the result in (49c) into the context of (47):

(50) \[ \lambda x \lambda \pi \lambda P \lambda u (\lambda w \lambda \pi_3 \exists z (\text{heavy}(z, \pi_3) \land o\text{-elab}(z, w, \pi_3)) \land (\pi * \text{ARG}^\text{book}_1: \text{P} \bullet \text{I} * \text{ARG}^\text{heavy}_1: \text{P})(u) \land P(\pi * \text{ARG}^\text{book}_1: \text{P} \bullet \text{I})(u))(x)(\lambda v \lambda \pi' \text{book}(v, \pi')) \]
Finishing up

Eliminating the higher order variable from (50) we get:

(51) a. \( \lambda x \lambda \pi \lambda u (\lambda w \lambda \pi \lambda z (\text{heavy}(z, \pi) \land \text{o-elab}(z, w, \pi))(\pi^*) \)
\( \text{ARG}_1^{\text{book}}: P \bullet I^* \text{ARG}_1^{\text{heavy}}: P)(u) \land \lambda v \lambda \pi' \text{book}(v, \pi')[\pi^* \)
\( \text{ARG}_1^{\text{book}}: P \bullet I^1(u)(x) \)

b. Via Application and Substitution:
\( \lambda x \lambda \pi \lambda u (\lambda w \lambda \pi \lambda z (\text{heavy}(z, \pi) \land \text{o-elab}(z, w, \pi))(\pi^*) \)
\( \text{ARG}_1^{\text{book}}: P \bullet I^* \text{ARG}_1^{\text{heavy}}: P)(u) \land \lambda v \text{book}(v, \pi^*) \)
\( \text{ARG}_1^{\text{book}}: P \bullet I^1(u)(x) \)

c. Via Binding and Substitution:
\( \lambda x \lambda \pi \lambda u (\lambda w \lambda \pi \lambda z (\text{heavy}(z, \pi) \land \text{o-elab}(z, w, \pi))(\pi^*) \)
\( (\pi^* \text{ARG}_1^{\text{book}}: P \bullet I^* \text{ARG}_1^{\text{heavy}}: P)[u] \land \lambda v \text{book}(v, \pi)[u]][x] \)

d. Again using Application and Substitution (four times):
\( \lambda x \lambda \pi (\lambda z (\pi \lambda z (\text{heavy}(z, \pi) \land \text{o-elab}(z, x, \pi)))) \)
\( (\pi^* \text{ARG}_1^{\text{book}}: P \bullet I^* \text{ARG}_1^{\text{heavy}}: P)[x] \land \text{book}(x, \pi) \)

e. Further Applications and Substitutions
\( \lambda x \lambda \pi (\exists z (\text{heavy}(z, \pi \lambda z (\text{heavy}(z, \pi) \land \text{o-elab}(z, x, \pi))); \pi^*) \)
\( \land \text{o-elab}(z, x, \pi \lambda z (\text{heavy}(z, \pi) \land \text{o-elab}(z, x, \pi)); \pi^*) \)
\( \land \text{book}(x, \pi)) \)

f. i.e.: \( \lambda x: P \bullet I \lambda \pi \lambda z: P (\text{heavy}(z, \pi) \land \text{o-elab}(z, x, \pi) \land \text{book}(x, \pi)) \)

We can now apply the determiner meaning to (50) and get our completed logical form for \textit{a heavy book}:

(52) \( \lambda Q \lambda \pi \exists u: P \bullet I \neg \exists z: (\text{heavy}(z, \pi) \land \text{o-elab}(z, u, \pi) \land \text{book}(u, \pi) \land Q(\pi)(x)) \)
Generalization: Presupposition justification rules

Definition 8 Justifying a ● type:

\[ \phi(v, \pi), \pi \text{ carries } \text{ARG}_i^p: \alpha \bullet \beta \ast \text{ARG}_j^q: \alpha / \beta, \quad v \in \text{ARG}_i^p \cap \text{ARG}_j^q \]

To define justification of a presupposition of a constituent type on a term that features a ● type argument, we apply \( B \) which takes us “back” from a property of objects of ● type to a property of objects of one of its constituent types.

Definition 9 Justifying a constituent type of a ● type:

\[ \phi(v, \pi), \pi \text{ carries } \text{ARG}_i^q: \alpha / \beta \ast \text{ARG}_j^p: \alpha \bullet \beta \quad v \in \text{ARG}_i^p \cap \text{ARG}_j^q \]

\[ B(\lambda w \lambda \pi_1 \phi(w, \pi_1))(\pi)(v) \]

These justification rules say that when there is a type conflict involving a ● type, we can justify the relevant type requirement by applying functors that allow for predications to aspects of objects of complex type.

Anaphors

(53) John's mother burned the book on magic before he mastered it.

(54) \( \lambda \pi \) before(∃\( y( \text{mom}(y, j, \pi) \wedge \exists x \exists z(\text{book}(z, \pi) \wedge \text{o-elab}(z, x, \pi) \wedge \text{burn}(y, x, \pi))), \exists u \exists v: i \ (u = ?(\pi) \wedge v = ?(\pi) \wedge \text{master}(u, v, \pi))) \)
Quantificational Puzzles

(55)  a. The student read every book in the library.
     b. The student carried off every book in the library.

(56)  \[ \lambda \pi' \exists w \ (\text{student}(w) \land \forall v \exists u \exists z \exists x \ (\text{library}(x, \pi') \land \text{in}(u, z, \pi') \land \text{o-elab}(z, x, \pi') \land \text{book}(v, \pi') \land \text{o-elab}(u, v, \pi')) \rightarrow \text{read}(w, v, \pi')) \]

(57)  \[ \lambda \pi' \exists y \ (\text{student}(y) \land \forall v \exists u \exists z \exists x \ (\text{library}(x, \pi') \land \text{in}(u, z, \pi') \land \text{o-elab}(z, x, \pi') \land \text{book}(w, \pi') \land \text{o-elab}(u, w, \pi') \land \text{o-elab}(v, w, \pi')) \rightarrow \text{carry off}(y, v, \pi')) \]

Copredication revisited

We now return to the copredication examples. Let us look at a classic example of copredication.

(58)  John picked up and mastered three books.

- Coordination Rule for \textit{and}:
  - Given a \( \lambda \) term of the following form
    \[ [\lambda \phi \lambda \psi \lambda \pi \lambda \psi_1 \lambda \pi_1 \Phi(Pre_{\phi_1}(\pi)))(\lambda u \lambda \pi_1 \Phi(Pre_{\phi_2}(\pi)))(\psi)] \]
    and
    \[ [\lambda \phi \lambda \psi \lambda \pi \lambda \psi_1 \lambda \pi_1 \Phi(Pre_{\psi_1}(\pi)))(\lambda u \lambda \pi_1 \Phi(Pre_{\psi_2}(\pi)))(\psi)], \]
  - one may rewrite the construction as the following \( \lambda \) term:
    \[ \lambda \psi_1 \lambda \psi_2 \lambda \pi_1 \lambda \pi_2 \lambda \phi(Pre_{\phi_1}(\pi)))(\lambda u \lambda \pi_1 \Phi(Pre_{\phi_2}(\pi)))(\lambda x \lambda \pi_2(\phi(\pi_2)(x)(u) \land \psi(\pi_2)(x)(u))) \]
Where to justify presuppositions

Normally where they are carried to (local justification)

- Local Presupposition Justification in Coordinated Predicative Constructions:
  Suppose $\text{Pre}_\psi(\text{Pre}_\phi(\pi))$ is unsatisfiable. Then:
  \[
  \Phi(\text{Pre}_\psi(\text{Pre}_\phi(\pi)))(\lambda x \lambda \pi_1(\phi(\pi_1)(x) \land \psi(\pi_1)(x)))
  \]

The finished logical form

(59) $\lambda \pi \exists u (u = j(\pi) \land \exists w: \text{P} \land \exists z: \text{P} \land \text{pick-up}(u, z, \pi) \land \text{o-elab}(z, w, \pi) \land \exists z': \text{I} \land \text{master}(u, z', \pi))$

This same strategy can be used on modifiers as well as verbs and VPs. For instance, consider the coordinated modifiers in (61):

(61) a lengthy but fascinating novel

(59) John picked up and mastered three books.

(60) $\lambda \pi \exists u (u = j(\pi) \land \exists w: \text{P} \land \exists z: \text{P} \land \text{pick-up}(u, z, \pi) \land \text{o-elab}(z, w, \pi) \land \exists z': \text{I} \land \text{master}(u, z', \pi))$