# Semantik 7. Quantoren: Semantischer Typ

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# Quantifiers

## (1) Some quantified DPs:

- a. Kein Mensch ist illegal.
- b. Alle Bücher im Regal sind alt.
- c. Ich kenne jedes Buch von Chomsky.
- d. Maria hat ein schönes Fahrrad.
- e. Sie mag nichts.
- f. Alles tut weh.
- g. Er hat (et)was gestohlen.

## Two initial possibilities:

- ▶ The denotation of quantified DPs is  $\in D_e$ .
- ▶ The denotation of quantified DPs is  $\in D_{\langle e,t \rangle}$ .

#### Problem:

Neither of these approaches is correct. The denotation of quantified DPs must be of a different type.

# Wrong Predictions about Truth-Conditions and Entailment Patterns 1: Subset-to-Superset Inferences

First try:

The denotation of quantified DPs is  $\in D_e$ .

Note:

There are quantified DPs that fail to validate subset-to-superset inferences.

- (2) A valid inference with  $D_e$  type subjects:
  - a. John came yesterday morning
  - b. ⇒John came yesterday

This inference follows under the following three standard assumptions:

- (3) Assumptions:
  - $\mathsf{a}.\quad [\![John]\!]\in \mathsf{D}_e.$
  - $b. \quad [\![ came \ yesterday \ morning]\!] \subseteq [\![ came \ yesterday]\!]$
  - c. A sentence whose subject denotes an individual is true iff that individual is a member of the set denoted by the VP.

## (4) An invalid inference with quantified subjects:

- a. At most one/no letter came yesterday morning
- b.  $\Rightarrow$ At most one/no letter came yesterday

#### Conclusion:

If assumptions (3-b) and (3-c) are to be maintained, then assumption (3-a) must be given up for quantified DPs like **at most one letter**, **no letter**, **few letters**, etc.

# Wrong Predictions about Truth-Conditions and Entailment Patterns 2: Law of Contradiction

#### Note:

There are DPs that fail the Law of Contradiction

- (5) The Law of Contradiction:  $\neg [p \& \neg p]$
- A contradiction with D<sub>e</sub> type subjects:
  Mount Rainier is on this side of the border, and Mount Rainier is on the other side of the border

## (7) Assumptions:

- a.  $[\![Mount Rainier]\!] \in D_e.$
- b.  $[\![be on this side of the border]\!] \cap [\![be on the other side of the border]\!] = \{ \}.$
- c. A sentence whose subject denotes an individual is true iff that individual is a member of the set denoted by the VP.
- d. We have a standard analysis of  $\mathbf{and}$ .

#### (8) A contingent statement with quantified subjects:

- a. More than two mountains are on this side of the border, and more than two mountains are on the other side of the border
- b. A mountain is on this side of the border, and a mountain is on the other side of the border

# Wrong Predictions about Truth-Conditions and Entailment Patterns 3: Excluded Middle

#### Note:

There are DPs that fail the Law of Excluded Middle.

- (9) The Law of Excluded Middle: p or ¬p
- (10) A tautology with  $D_e$  type subjects: I am over 30 years old, or I am under 40 years old
- (11) Assumptions:
  - $\mathsf{a}.\quad [\![\mathbf{I}]\!]\in\mathsf{D}_e.$
  - $b. \quad [\![ \text{be over 30 years old}]\!] \cup [\![ \text{be under 40 years old}]\!] = \mathsf{D}_e.$
  - c. A sentence whose subject denotes an individual is true iff that individual is a member of the set denoted by the VP.
  - d. We have a standard analysis of **or**.

#### (12) A contingent statement with quantified subjects: Every woman in this room is over 30 years old, or every woman in this room is under 40 years old

(Relevant context that makes (12) false: One woman in this room is under 30 years old, and one woman is over 40 years old.)

# Wrong Predictions about Scope Ambiguity and the Effects of Syntactic Reorganization

#### (13) Identical truth-conditions after topicalization of DPs of type e:

- a. I answered [ question no. 7 ]<sub>1</sub>
- b. [Question no. 7] wh<sub>1</sub> I answered  $t_1$
- (14) Identical truth-conditions with "such that" constructions involving DPs of type e:
  - a. John saw Mary
  - b. Mary is such<sub>2</sub> that John saw her<sub>2</sub>
  - c. John is such<sub>1</sub> that he<sub>1</sub> saw Mary (N.B.: misprint in the book, p. 135)

This follows under present assumptions, from the Predicate Abstraction rule, given that topicalization involves an invisible wh-pronoun that permits the application of the rule. (Actually, this will turn out to be irrelevant in the next chapter: only the index is interpreted, and we don't really need the wh-pronoun to feed PA).

(15) Example (14-b) – same meaning as (14-a):

- a.  $\llbracket Mary \text{ is such}_2 \text{ that John saw her}_2 \rrbracket = (by FA)$
- b.  $[\![is such_2 that John saw her_2]\!]([\![Mary]\!]) = (by vacuity of is, TN and PA)$

c. [ 
$$\lambda x \in D_e$$
 . [[that John saw her<sub>2</sub>]]<sup>[2 \to x]</sup>] (Mary)

- = (by vacuity of**that**and FA)
- d.  $[\lambda x \in D_e . [saw]^{[2 \to x]}([her_2]^{[2 \to x]})([John]^{[2 \to x]})]$  (Mary) = (by Irrelevance of Assignments, TN, and Traces and Pronouns Rule)

e. [
$$\lambda x \in D_e$$
. [ $\lambda y \in D_e$ . [ $\lambda z \in D_e$ . z saw y ]] (x)(John) ]  
(Mary)

(by  $\lambda$ -conversion)

$$\begin{split} \text{f.} & \left[ \begin{array}{l} \lambda \mathsf{x} \in \mathsf{D}_e \ . \ \text{John saw } \mathsf{x} \end{array} \right] \text{(Mary)} \\ & = 1 \text{ iff John saw Mary} \end{split}$$

# Semantically Non-Vacuous Movement

- (16) Non-identical truth-conditions after topicalization of quantified DPs:
  - Almost everybody<sub>1</sub> answered at least one question<sub>2</sub>
    "For almost everybody, there is at least one question that (s)he answered."
  - b. At least one question<sub>2</sub> almost everybody<sub>1</sub> answered t<sub>2</sub> "There is at least one question that was answered by almost everybody."

#### Disambiguating context:

10 students (S), 10 questions (Q). S 1 answered just Q 1, S 2 answered just Q 2, and so on, except for S 10, who did not answer a question Q. In that case, (16-a) is true, and (16-b) is false. (N.B.: (16-b) logically implies (16-a); if (16-b) is true, (16-a) is also true: If there is one question that was answered by almost everyone, it follows that for almost everyone, there is a question that (s)he answered – it just happens to be the same question for all.)

# Semantically Non-Vacuous Such-That-Constructions

(17) Non-identical truth-conditions with "such that" constructions involving quantified DPs:

- a. Nobody<sub>1</sub> saw more than one policeman<sub>2</sub>
- b. More than one policeman is  $such_2$  that  $nobody_1$  saw  $him_2$
- c. Nobody is  $such_1$  that  $(s)he_1$  saw more than one policeman

#### Disambiguating contexts:

10 policemen (P), 10 people (W) who look for them. First, suppose that W 1 saw just P 1, W 2 saw just P 2, and so on, and W 10 saw just P 10. In that case, (17-a,c) are true, and (17-b) is false. Second, suppose that P 1 and P 2 were not seen at all, and that W 1 actually saw P 3 and P 4. In that case, (17-b) is true, and (17-a,c) are false.

#### (18) Negation, DPs of type e, and quantified DPs:

- a. It did not snow on Christmas day
- b. It did not snow on more than two of these days
  - "It is not the case that it snowed on more than two of these days."
  - "On more than two of these days was it the case that it did not snow."

(unambiguous)

(ambiguous):

#### Disambiguating context:

First, suppose that there are 7 days, and that there was snow on exactly 3 days. In that case, (18-b) is true on reading (ii), and false on reading (i). Second, suppose that there are 3 days, and that there was snow on exactly 2 days. In that case, (18-b) is true on reading (i), and false on reading (ii).

- (19) Hutkontur (I-topicalization) pattern in German:
  - a. An mehr als drei Tagen hat es nicht geschneit
  - b. An MEHR als drei Tagen hat es NICHT geschneit

#### (20) More Hutkontur:

- a. Alle Politiker sind nicht korrupt
- b. Alle Politiker sind nicht korrupt

#### Conclusion:

These effects are completely unexpected if quantified DPs have the same type as, e.g., proper names. Due to the way PA works, movement of entities with a denotation in  $D_e$  and "such that" constructions involving entities with a denotation in  $D_e$  cannot affect truth-conditions. Since truth-conditions are affected in the case of quantified DPs, we can conclude that they are not of type e.

Second try: The denotation of quantified DPs is  $\in D_{\langle e,t \rangle}$ .

# Quantified DPs as Denoting Sets of Individuals

## Exercise, p. 138

## (21) Lexical entries:

a. 
$$[Ann] = {Ann}$$

b. 
$$\llbracket Jacob \rrbracket = \{Jacob\}$$

- c. [[everything]] = D
- d.  $[[nothing]] = \{ \}$
- e.  $\llbracket vanished \rrbracket = \lambda X \in Pow(D) . X \subseteq \{y \in D . y vanished\}$
- $f. \quad [\![ \textbf{reappeared} ]\!] = \lambda X \in \mathsf{Pow}(\mathsf{D}) \ . \ X \subseteq \{ \mathsf{y} \in \mathsf{D} \ . \ \mathsf{y} \\ \mathsf{reappeared} \}$

# (a) (22)

#### Proper names:

$$\label{eq:started} \begin{split} & \llbracket \textbf{Ann vanished} \rrbracket = 1 \text{ iff} & (by FA) \\ & \llbracket \textbf{vanished} \rrbracket (\llbracket \textbf{Ann} \rrbracket) = 1 \text{ iff} & (by TN) \\ & \lambda X \in \mathsf{Pow}(\mathsf{D}) \ . \ X \subseteq \{ y \in \mathsf{D} \ . \ y \text{ vanished} \} (\{\mathsf{Ann}\}) = 1 \text{ iff} \\ & \{\mathsf{Ann}\} \subseteq \{ y \in \mathsf{D} \ . \ y \text{ vanished} \} & \Rightarrow \text{ correct result} \end{split}$$

#### (23) Lexical entries:

- $\mathsf{a.}\quad [\![\textbf{Ann}]\!]=\{\mathsf{Ann}\}$
- b.  $[[Jacob]] = {Jacob}$
- c. [[everything]] = D
- d.  $[[nothing]] = \{ \}$
- e.  $\llbracket vanished \rrbracket = \lambda X \in Pow(D) . X \subseteq \{y \in D . y vanished\}$
- $f. \quad [\![ \textbf{reappeared} ]\!] = \lambda X \in \mathsf{Pow}(\mathsf{D}) \ . \ X \subseteq \{ \mathsf{y} \in \mathsf{D} \ . \ \mathsf{y} \\ \mathsf{reappeared} \}$

## (25) Lexical entries:

- $\mathsf{a.}\quad [\![\textbf{Ann}]\!]=\{\mathsf{Ann}\}$
- b.  $\llbracket Jacob \rrbracket = \{Jacob\}$
- c. [[everything]] = D
- $\mathsf{d}.\quad [\![\textbf{nothing}]\!]=\{\ \}$
- e.  $\llbracket vanished \rrbracket = \lambda X \in Pow(D) . X \subseteq \{y \in D . y vanished\}$
- $\label{eq:f.second} \begin{array}{ll} f. & [\![ \textbf{reappeared} ]\!] = \lambda X \in \mathsf{Pow}(\mathsf{D}) \ . \ X \subseteq \{ \mathsf{y} \in \mathsf{D} \ . \ \mathsf{y} \\ & \mathsf{reappeared} \} \end{array}$
- (26) Negative quantifier: [nothing vanished]] = 1 iff (by FA) [vanished]]([nothing]]) = 1 iff (by TN)  $\lambda X \in Pow(D) . X \subseteq \{y \in D . y \text{ vanished}\}(\{ \}) = 1 \text{ iff}$   $\{ \} \subseteq \{y \in D . y \text{ vanished}\}$ Since, for all sets A,  $\{ \} \subseteq A$ , this sentence is predicted to be a tautology. However, it is clearly false if there is something that vanished. Hence:  $\Rightarrow$  wrong result

To solve this problem, we might change the lexical entry of vanished as follows:

(27) 
$$\llbracket vanished_2 \rrbracket = \lambda X \in Pow(D) . X = \{y \in D . y vanished\}$$

Now the truth-conditions for universal and negative quantification are correctly predicted, but the proper name case does not work anymore: **[Ann vanished]** would be predicted to be true iff the set of vanishing individuals is identical to the singleton set containing Ann (i.e., no-one else could have vanished).

Furthermore, it is hard to see how this proposal can be extended to other quantifier phrases, like, e.g., **something** or **some men**. Which set should **some men** denote? It must be some subset of  $D_e$ ; of course, it cannot always be the same subset, but must be able to vary. However, it must not vary in the following inference (otherwise, this inference would be predicted to be invalid, which it obviously is not):

#### (28) A valid inference:

- a. Some men are fools
- b. All fools are unhappy
- c.  $\Rightarrow$  Some men are unhappy

(cf. John is a fool)

(cf. John is unhappy)

Thus, **some men** must refer to the same class in (28-a) and (28-c). However, this assumption leads to an incorrect prediction concerning the invalid inference in (29) (it should be valid as well):

## (29) An invalid inference:

- a. Some men are fools (cf. John, Mary, and Bill are fools)
- b. Some men are unhappy (cf. John, Mary, and Bill are unhappy)
- c. *⇒*Some men are unhappy fools (cf. John, Mary, and Bill are unhappy fools)

## Conclusion (Geach (1972, 57)):

The idea that quantified DPs like **some men** denote sets of individuals must be wrong.

## Second Part of the Exercise on p. 139

(b)

## "Something," "nothing," "everything"

- (31) a. Nothing vanished
  - b. Everything vanished
  - c. Something vanished

It has turned out to be impossible to assume that quantified DPs are of type e (or of type  $\langle e,t \rangle$ ). If we maintain the idea that VP denotations are of type  $\langle e,t \rangle$ , there is one obvious further possibility: Quantified DPs could have  $\boxed{\langle \langle e,t \rangle,t \rangle}$  type denotations, i.e., they could take VP denotations as arguments. This is indeed the correct solution. Quantified DPs ("generalized quantifiers") are functions from  $\langle e,t \rangle$  to t, i.e., characteristic functions of sets of properties (properties = sets of individuals), i.e., "second-order properties."

## (32) Some generalized quantifiers:

- a. [[nothing]] = λf ∈ D<sub><e,t></sub> . ¬∃x ∈ D<sub>e</sub>: f(x) = 1
  = the set of predicates that are true of nothing (or the set of properties that nothing has)
- b.  $[[something]] = \lambda f \in D_{\langle e,t \rangle}$ .  $\exists x \in D_e$ : f(x) = 1= the set of predicates that are true of something (or the set of properties that something has)
- c.  $\llbracket everything \rrbracket = \lambda f \in D_{\langle e,t \rangle}$ .  $\forall x \in D_e$ : f(x) = 1= the set of predicates that are true of everything (or the set of properties that everything has)

This means that, depending on the type of the subject, VP can either be an argument (of a generalized quantifier of type <<e,t>,t>), or a function (that takes a subject with an denotation of type e as an argument).

(33) a. Nothing vanished b. Mary vanished  $[\![\text{nothing}]\!] \in \mathsf{D}_{<< e,t>,t>} \\ [\![\text{Mary}]\!] \in \mathsf{D}_e$ 

#### Exercise, p. 142

e. 
$$\neg \exists x \in D_e$$
: x vanished

(Similarly for something, everything.)

## (i) Subset to Superset

- (35) An invalid inference with quantified subjects:
  - a. Nothing happened yesterday morning
  - b. **An Antiperiod Sector**

It must be shown that a situation is possible where, e.g., **[nothing]**(**[happened yesterday morning]**) = 1, whereas **[nothing]**(**[happened yesterday]**) = 0. This is straightforward. Suppose that **[[happened yesterday morning]** = the characteristic function of the empty set, whereas **[[happened yesterday]**] is the characteristic function of a non-empty set (a couple of things happened later in the day). In that case, **[**(35-a)]] = 1, and **[**(35-b)]] = 0.

## (ii) Law of Contradiction

(36) A contingent statement with quantified subjects:A mountain is on this side of the border, and a mountain is on the other side of the border

It must be shown that a situation is possible where a generalized quantifier f (such as **[a mountain]**) yields 1 if it is applied to a predicate denotation like **[on this side of the border]**, and also 1 if it is applied to a predicate denotation like **[on the other side of the border]**, so that (36) can be true. Such a situation occurs if there is a mountain that is on this side of the border, and if there is another mountain that is on the other side of the border.

#### (iii) Law of Excluded Middle

(37) A contingent statement with quantified subjects:
 Every woman in this room is over 30 years old, or every woman in this room is under 40 years old

It must be shown that a situation is possible where a generalized quantifier f (such as **[every woman in this room**]) yields 0 if it is applied to **[is over 30 years old**], and also yields 0 if it is applied to **[is under 40 years old**], so that (37) can be false. Such a situation occurs if it is not the case that all women in this room are over 30 years old (there is at least one woman who is younger), and if it is not the case that they are all under 40 years old (there is at least one woman who is older).

# Problems Avoided: Scope Ambiguity & Syntactic Reorganization

(iv) Scope Ambiguity & Syntactic Reorganization (See chapter 7.)

# Generalized Quantifiers vs. Quantifying Determiners

**everything**, **something**, **nothing** are generalized quantifiers with denotations in  $D_{<<e,t>,t>}$ . The same conclusion holds with complex quantifier phrases such as **every painting**, **some painting**, **a painting**, **no painting**:

- (38) a. Every painting vanished
  - b. Some painting vanished

(= A painting vanished)

c. No painting vanished

#### Note:

(i)  $\llbracket painting \rrbracket \in D_{<e,t>}$ (Cf. This is a painting which 1 John likes  $t_1$ , or the painting) (ii)  $\llbracket every \ painting \rrbracket \in D_{<<e,t>,t>}$ (this generalized quantifier takes a VP denotation as its argument and assigns to it a truth-value) (iii) Hence, if D and NP are to combine via FA, we can conclude that quantifying determiners like every (some, no, etc.) have denotations in  $D_{<<e,t>,<<e,t>,t>>}$ .

## Lexical Entries for Quantifying Determiners

Consequently, we can adopt the following lexical entries:

(39) Some lexical entries for quantifying determiners:

- a.  $\llbracket every \rrbracket = [\lambda f \in D_{\langle e,t \rangle} . [\lambda g \in D_{\langle e,t \rangle} . \forall x \in D_e: f(x) = 1 \rightarrow g(x) = 1 ]]$
- b.  $[som_{e}] = [\lambda f \in D_{\langle e,t \rangle} . [\lambda g \in D_{\langle e,t \rangle} . \exists x \in D_{e}: f(x) = 1 \& g(x) = 1]]$

c. 
$$\llbracket a \rrbracket = \llbracket some \rrbracket$$
  
d.  $\llbracket no \rrbracket = \llbracket \lambda f \in D_{\leq e,t >}$ .  $\llbracket \lambda g \in D_{\leq e,t >}$ .  $\neg \exists x \in D_e$ :  $f(x) = 1 \& g(x) = 1$   
1 ]]

#### Terminology: Restriction vs. Nuclear Scope:

Quantifying determiners have denotations that are comparable to those of transitive verbs (transitive prepositions, adjectives, nouns). The only difference is that quantifying determiners take two arguments with denotations in  $D_{<e,t>}$ , whereas transitive predicates take two arguments with denotations in  $D_e$ . The first argument of a quantifying determiner (comparable to the object of a transitive verb) is often referred to as the restriction of the quantifying determiner (by combining the determiner and its restriction via FA, a generalized quantifier is created); the second argument of a quantifying determiner (comparable to the subject of a transitive verb) is often called the nuclear scope of the quantifying determiner.

#### Question:

What about non-complex generalized quantifiers like **everything**, **something**, **nothing**? They might have an implicit restriction of the type "is a thing." Similarly, and perhaps more to the point, we could assume implicit restrictions for **everyone**, **no-one**, **someone**, etc.:

$$\begin{array}{ll} \textbf{(40)} & \llbracket \textbf{everyone} \rrbracket = \\ \textbf{a.} & \llbracket \textbf{every} \rrbracket (\llbracket \textbf{one} \rrbracket) = \\ \textbf{b.} & \begin{bmatrix} \lambda f \in \mathsf{D}_{< e, t >} \ . \ [ \ \lambda g \in \mathsf{D}_{< e, t >} \ . \ \forall x \in \mathsf{D}_e \text{: } f(x) = 1 \rightarrow g(x) \\ & = 1 \end{bmatrix} ] & (\lambda y \in \mathsf{D}_e \ . \ y \text{ is a person}) = \\ \textbf{c.} & \begin{bmatrix} \lambda g \in \mathsf{D}_{< e, t >} \ . \ \forall x \in \mathsf{D}_e \text{: } x \text{ is a person} \rightarrow g(x) = 1 \end{bmatrix} \end{array}$$

# Calculating Truth Conditions

#### Exercise, p. 147

(41)**[No painting vanished]** = 1 iff (by FA) a. [no painting]([vanished]) = 1 iff (by FA) b. c. [no]([painting])([vanished]) = 1 iff (by TN) d. [ $\lambda f \in D_{\langle e,t \rangle}$ .]  $\lambda g \in D_{\langle e,t \rangle}$ .  $\neg \exists x \in D_e$ : f(x) = 1 & g(x) $= 1 ]] (\lambda z \in D_e . z \text{ is a painting}) (\lambda y \in D_e . y \text{ vanished}) =$ 1 iff (by  $\lambda$ -conversion) e. [ $\lambda g \in D_{\langle e,t \rangle}$ .  $\neg \exists x \in D_e$ : [ $\lambda z \in D_e$ . z is a painting ] (x)  $= 1 \& g(x) = 1 ] (\lambda y \in D_e$ . y vanished) = 1 iff (by  $\lambda$ -conversion) f. [ $\lambda g \in D_{\langle e,t \rangle}$ .  $\neg \exists x \in D_e$ : x is a painting & g(x) = 1 ] ( $\lambda y$  $\in D_e$ . y vanished) = 1 iff (by  $\lambda$ -conversion) g.  $\neg \exists x \in D_e$ : x is a painting & [ $\lambda y \in D_e$ . y vanished ] (x) = 1 iff (by  $\lambda$ -conversion) h.  $\neg \exists x \in D_e$ : x is a painting & x vanished

(Similarly for every painting, some painting.)