Semantik
8. Quantoren: Semantischer Typ 2

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Schönfinkeled and Relational Denotations for Transitive Verbs

Recall:

- (Right-to-left) Schönfinkeled denotations of transitive verbs are necessary, given that
  - semantic interpretation works via functional application, and
  - syntactically verbs combine with objects first, with subjects later.
- From a purely semantic perspective, Schönfinkeled denotations and corresponding relational denotations are normally equivalent.

(1) Schönfinkeled denotation of a transitive verb:
$[\![\textbf{mag}]\!] = \lambda \mathrm{y} \in \mathrm{D}\ .\ [\ \lambda \mathrm{x} \in \mathrm{D}\ .\ [\ \mathrm{x\ mag\ y}\ ]]$

(2) Relational denotation of a transitive verb:
$[\![\textbf{mag}]\!] = \{<\mathrm{x,y}> \in \mathrm{D}\times\mathrm{D}\text{: x mag y}\}$

Schönfinkeled and Relational Denotations for Determiners

In the present system, quantifying determiners are treated like transitive verbs; their denotations are Schönfinkeled functions from $D_{<e,t>}$ to functions from $D_{<e,t>}$ to truth-values. Like transitive verbs, quantifying determiners can therefore also be viewed as relations – not as simple first-order relations (relations between individuals), but as second-order relations (relations between sets of individuals, i.e., between properties). Hence, the functional, Schönfinkeled denotations of quantifying determiners adopted so far (cf. (3)) can be reformulated in a purely relational way (cf. (4)):

(3) Some lexical entries for quantifying determiners:

a. $[\![\textbf{every}]\!] = [\ \lambda f \in D_{<e,t>}\ .\ [\ \lambda g \in D_{<e,t>}\ .\ \forall x \in D_e\text{: } f(x) = 1 \rightarrow g(x) = 1\ ]]$
b. $[\![\textbf{some}]\!] = [\ \lambda f \in D_{<e,t>}\ .\ [\ \lambda g \in D_{<e,t>}\ .\ \exists x \in D_e\text{: } f(x) = 1\ \&\ g(x) = 1\ ]]$
c. $[\![\textbf{a}]\!] = [\![\textbf{some}]\!]$
d. $[\![\textbf{no}]\!] = [\ \lambda f \in D_{<e,t>}\ .\ [\ \lambda g \in D_{<e,t>}\ .\ \neg\exists x \in D_e\text{: } f(x) = 1\ \&\ g(x) = 1\ ]]$

(4) Some lexical entries for quantifying determiners, relational notation:

a. $[\![\textbf{every}]\!] = \{<A,B> \in Pow(D) \times Pow(D)\text{: } A \subseteq B\}$
b. $[\![\textbf{some}]\!] = \{<A,B> \in Pow(D) \times Pow(D)\text{: } A \cap B \neq \{\ \}\ \}$
c. $[\![\textbf{no}]\!] = \{<A,B> \in Pow(D) \times Pow(D)\text{: } A \cap B = \{\ \}\ \}$
d. $[\![\textbf{at least two}]\!] = \{<A,B> \in Pow(D) \times Pow(D)\text{: } |A \cap B| \geq 2\ \}$
e. $[\![\textbf{most}]\!] = \{<A,B> \in Pow(D) \times Pow(D)\text{: } |A \cap B| > |A - B|\ \}$
f. $[\![\textbf{few}]\!] = \{<A,B> \in Pow(D) \times Pow(D)\text{: } |A - B| > |A \cap B|\ \}$
etc.

From Relation to Schönfinkel

Note:
Of course, if we dispense with Schönfinkeled determiner denotations in favour of a strictly relational approach, new semantic composition principles must be introduced, because then, functional application (FA) alone will not suffice anymore. Otherwise, the two approaches (and many other "intermediate" approaches that arise due to the possible choices between sets and characteristic functions, and between various options of Schönfinkelization) are equivalent.

(5) How to derive a Schönfinkeled functional denotation from a relation:

a. $R_{every} = \{<A,B> \in Pow(D) \times Pow(D)\text{: } A \subseteq B\}$
(by turning the set into a characteristic function)

b. $F_{every} = \lambda <A,B> \in Pow(D) \times Pow(D)\ .\ A \subseteq B$
(by Schönfinkelization from left to right – note that this is because the determiner combines first with the NP denotation, and then with the VP denotation)

c. $f_{every} = [\ \lambda A \in Pow(D)\ .\ [\ \lambda B \in Pow(D)\ .\ A \subseteq B\ ]]$
(by turning the sets A, B into their characteristic functions)

d. $f'_{every} = [\ \lambda f \in D_{<e,t>}\ .\ [\ \lambda g \in D_{<e,t>}\ .\ \{x \in D_e\text{: } f(x) = 1\} \subseteq \{y \in D_e\text{: } g(y) = 1\}$
(by definition of subset relation)

e. $f''_{every} = [\ \lambda f \in D_{<e,t>}\ .\ [\ \lambda g \in D_{<e,t>}\ .\ \forall x \in D_e\text{: } f(x) = 1 \rightarrow g(x) = 1\ ]]$
(= the original denotation for **every** in (3-a))

Another Option

Exercise, p. 150

Quantifying determiners as functions from Pow(D) to Pow(Pow(D)) (Barwise & Cooper (1981)):

(a)

(6) Some lexical entries:

a. $[\![\textbf{every}]\!] = \lambda A \in Pow(D)\ .\ \{B \in Pow(D)\text{: } A \subseteq B\}$

b. $[\![\textbf{some}]\!] = \lambda A \in Pow(D)\ .\ \{B \in Pow(D)\text{: } A \cap B \neq \{\ \}\ \}$

c. $[\![\textbf{no}]\!] = \lambda A \in Pow(D)\ .\ \{B \in Pow(D)\text{: } A \cap B = \{\ \}\ \}$

(7) Suppose:

a. $D = \{a,b,c\}$. Then:

b. $Pow(D) = \{\ \{\ \}, \{a\}, \{b\}, \{c\}, \{a,b\}, \{a,c\}, \{b,c\}, \{a,b,c\}\ \}$

c. $Pow(Pow(D)) = \{\ \{\ \}, \{\{\ \}\}, \{\{\ \},\{a\}\}, \{\{a\},\{a,b\}\},$
$\{\{a\},\{a,b\},\{a,b,c\}\}, \{\{b\},\{c\},\{a,b\},\{a,c\}\}, \ldots\ \}$ ($2^{12} = 4096$ members)

(8) Rules:

a. We need no special rule for $[\![DP]\!]$ ($= [\![D]\!]([\![NP]\!])$, by FA).

b. If $\alpha = S$, $\beta = DP$, and $\gamma = VP$, and β and γ are α's daughters, then:
$[\![S]\!] = 1$ iff either $[\![VP]\!] \in [\![DP]\!]$, or $[\![DP]\!] \in [\![VP]\!]$.

c. $[\![VP]\!]$ and $[\![NP]\!]$ denote sets of individuals.

Barwise and Cooper: Continuation

(c)

(9) A one-to-one correspondence between functions from Pow(D) into Pow(Pow(D)) and relations between subsets of D

a. $[\![$**every**$]\!]$ $=$

b. $\lambda A \in$ Pow(D) . $\{B \in$ Pow(D): $A \subseteq B\}$
(= function from Pow(D) into Pow(Pow(D)); by characteristic function of $\{B \in$ Pow(D): $A \subseteq B\}$) $=$

c. [$\lambda A \in$ Pow(D) . [$\lambda B \in$ Pow(D) . $A \subseteq B$]]
(by definition of Pow(D) = the sets of all subsets of D) $=$

d. [$\lambda A \subseteq D$. [$\lambda B \subseteq D$. $A \subseteq B$]]
(by De-Schönfinkelization, i.e., by creating a two-place function) $=$

e. λ <A,B> $\in$ Pow(D)$\times$Pow(D) . $A \subseteq B$
(by definition of characteristic function) $=$

f. $\{$<A,B> $\in$ Pow(D)$\times$Pow(D): $A \subseteq B$ $\}$

Formal Properties of Relational Determiner Denotations

(10) Some properties of relations that are relevant for quantifying determiners δ:

a. Reflexivity:
δ is reflexive iff for all A: $<$A,A$> \in R_\delta$

b. Irreflexivity:
δ is irreflexive iff for all A: $<$A,A$> \notin R_\delta$

c. Symmetry:
δ is symmetric iff for all A, B: If $<$A,B$> \in R_\delta$, then $<$B,A$> \in R_\delta$.

d. Antisymmetry:
δ is antisymmetric iff for all A, B: If $<$A,B$> \in R_\delta$, and $<$B,A$> \in R_\delta$, then A = B.

e. Transitivity:
δ is transitive iff for all A, B, C: If $<$A,B$> \in R_\delta$, and $<$B,C$> \in R_\delta$, then $<$A,C$> \in R_\delta$.

f. Conservativity (the "lives on"-property of Barwise & Cooper (1981)):
δ is conservative iff for all A, B: $<$A,B$> \in R_\delta$ iff $<$A,A$\cap$B$> \in R_\delta$

g. Left Upward Monotonicity (Persistency, Barwise & Cooper (1981)):
δ is left upward monotone iff for all A, B, C: If A $\subseteq$ B and $<$A,C$> \in R_\delta$, then $<$B,C$> \in R_\delta$

h. Left Downward Monotonicity (Antipersistency, Barwise & Cooper (1981)):
If A $\subseteq$ B and $<$B,C$> \in R_\delta$, then $<$A,C$> \in R_\delta$

i. Right Upward Monotonicity:
If A $\subseteq$ B and $<$C,A$> \in R_\delta$, then $<$C,B$> \in R_\delta$

j. Right Downward Monotonicity:
If A $\subseteq$ B and $<$C,B$> \in R_\delta$, then $<$C,A$> \in R_\delta$

Illustration of Relational Properties by Determiners

(Exercise, p. 152)

(11) Reflexivity:
δ is reflexive iff for all A: $<A,A> \in R_\delta$
⟦**every**⟧ is reflexive. ⟦**some**⟧ and ⟦**no**⟧ are not.
(**Every A is an A** is always true, **Some A is an A** is true if there is an individual which is in A, and false if there is no such individual.)

(12) Irreflexivity:
δ is irreflexive iff for all A: $<A,A> \notin R_\delta$
⟦**not all**⟧ is irreflexive. ⟦**every**⟧ and ⟦**some**⟧ are not.
(**Not all As are As** is always false, in contrast to **every**, **some**, which may or must be true in this context.)

(13) Symmetry:
δ is symmetric iff for all A, B: If $<A,B> \in R_\delta$, then $<B,A> \in R_\delta$.
⟦**some**⟧ is symmetric, ⟦**no**⟧ is symmetric, but ⟦**every**⟧, ⟦**most**⟧ are not.
(If **Some men are fools** is true, then **Some fools are men** is also true; if **No men are fools** is true, then **No fools are men** is also true; but if **Every man is a fool** is true, then **Every fool is a man** does not have to be true.)

More Illustration

(14) Antisymmetry:
δ is antisymmetric iff for all A, B: If $<A,B> \in R_\delta$, and $<B,A> \in R_\delta$, then $A = B$.
$[\![$**every**$]\!]$ is antisymmetric, $[\![$**some**$]\!]$, $[\![$**most**$]\!]$, and $[\![$**no**$]\!]$ are not.

(15) Transitivity:
δ is transitive iff for all A, B, C: If $<A,B> \in R_\delta$, and $<B,C> \in R_\delta$, then $<A,C> \in R_\delta$.
$[\![$**every**$]\!]$ is transitive, $[\![$**some**$]\!]$ and $[\![$**no**$]\!]$ are not.
(If **Every man is a fool** is true and **Every fool is boring** is true, then **Every man is boring** is also true; if **Some men are fools** is true and **Some fools are boring** is true, then **Some men are boring** does not have to be true (all the boring fools could be those that are not men). Similarly, if **No woman is a fool** is true and **No fool is boring** is true, then **No woman is boring** does not have to be true.)

(16) Conservativity (the "lives on"-property of Barwise & Cooper (1981)):
δ is conservative iff for all A, B: $<A,B> \in R_\delta$ iff $<A,A\cap B> \in R_\delta$
All quantifying determiners are conservative; $[\![$**only**$]\!]$ is not conservative.
(**Only Germans are Nazis**: If someone is a Nazi, he is German. $\neq$ **Only Germans are German Nazis**: If someone is a German Nazi, he is German. The latter is a tautology, the former is contingent.)

Yet More Illustration

(17) Left Upward Monotonicity (Persistency, Barwise & Cooper (1981)):
δ is left upward monotone iff for all A, B, C: If A $\subseteq$ B and $<$A,C$>$ $\in$ R_δ, then $<$B,C$>$ $\in$ R_δ
⟦**some**⟧ is left upward monotone, ⟦**every**⟧, ⟦**few**⟧, ⟦**no**⟧ are not.
(If **Some old men are stupid** is true, **Some men are stupid** is also true. If, e.g., **Every old man is stupid** is true, then **Every man is stupid** does not have to be.)

(18) Left Downward Monotonicity (Antipersistency, Barwise & Cooper (1981)):
If A $\subseteq$ B and $<$B,C$>$ $\in$ R_δ, then $<$A,C$>$ $\in$ R_δ
⟦**every**⟧ is left downward monotone, ⟦**some**⟧ is not.
(If **Every man is stupid** is true, **Every old man is stupid** is also true. If **Some men are stupid** is true, then **Some old men are stupid** does not have to be true.)

(19) Right Upward Monotonicity:
If A $\subseteq$ B and $<$C,A$>$ $\in$ R_δ, then $<$C,B$>$ $\in$ R_δ
⟦**some**⟧ and ⟦**every**⟧ are right upward monotone, ⟦**no**⟧ and ⟦**few**⟧ are not.
(If **Someone/Everyone slept well** is true, then **Someone/Everyone slept** is also true; if **No-one/Few people slept well** is true, **No-one/Few people slept** does not have to be true.)

(20) Right Downward Monotonicity:
If A $\subseteq$ B and $<$C,B$>$ $\in$ R_δ, then $<$C,A$>$ $\in$ R_δ
⟦**no**⟧ and ⟦**few**⟧ are right downward monotone, ⟦**some**⟧ and ⟦**every**⟧ are not.
(If **Someone/Everyone slept** is true, **Someone/Everyone slept well** does not have to be; if **No-one/Few people slept** is true, then **No-one/Few people slept well** is also true (at least on one reading of **few** (Barwise & Cooper 1981, 186)).

Exercise on ‘there’ Insertion, p. 152

(21) The original examples:

a. **There are some apples in my pocket**
b. ***There is every apple in my pocket**

(22) Further well-formed examples:

a. **There is an apple in my pocket**
b. **There are two apples in my pocket**
c. **There are many apples in my pocket**
d. **There are very few apples in my pocket**
e. **There are no apples in my pocket**

(23) Further ill-formed examples:

a. ***There are all apples in my pocket**
b. ***There are most apples in my pocket**
c. ***There are not all apples in my pocket**
d. ***There is the apple in my pocket**

Crucial property (informal):
Only indefinite determiners may show up in the “there” construction.
Milsark’s (1977) generalization:
Determiners are weak or strong; only weak determiniers may show up in the “there” construction.
Barwise & Cooper’s (1981) explanation of the weak/strong distinction:
A determiner is strong iff it is either reflexive (**every**, **most**, **all**), or irreflexive (**not all**). (Note: **the**, **the two** and other “presuppositional” determiners must also be classified as reflexive.) A determiner that is not strong is weak (**some, two, many, few, no**).

Exercise on Negative Polarity, p. 153

(24) The original examples:

a. **Very few people ever made it across the Cisnos range**
b. **Every friend of mine who ever visited Big Bend loved it**

(25) Further well-formed examples:

a. **At most ten students saw any deer**
b. **No student would ever do this**

(26) Ill-formed examples:

a. ***Some students saw any deer**
b. ***A student would ever do this**
c. ***The student would ever do this**
d. ***Many students saw any deer**

Correct generalization:
Only those determiners can license negative polarity items that are left downward monotone.

Negative Polarity: German

(27) Quantifying determiners that are left downward monotone:

a. **Alle Frauen rennen**. ⇒ **Alle Frauen mit roten Haaren rennen**.
b. **Jede Frauen rennt**. ⇒ **Jede Frau mit roten Haaren rennt**.
c. **Wenige Frauen rennen**. ⇒ **Wenige Frauen mit roten Haaren rennen**.
d. **Keine Frau rennt**. ⇒ **Keine Frau mit roten Haaren rennt**.

(28) Quantifying determiners that are not left downward monotone:

a. **Manche Frauen rennen**. ⇒ # **Manche Frauen mit roten Haaren rennen**.
b. **Die meisten Frauen rennen**. ⇒ # **Die meisten Frauen mit roten Haaren rennen**.
c. **Eine Frau rennt**. ⇒ # **Eine Frau mit roten Haaren rennt**. (Note: generic interpretation of eine must be excluded.)

(29) A negative polarity item:
***Ich habe auch nur irgendeinen Rest von Stolz**.

(30) Negative polarity licensing by quantifying determiners:

a. **Jeder Mann, der auch nur irgendeinen Rest von Stolz hatte, sagte das ab.**
b. **Kein Mann, der auch nur irgendeinen Rest von Stolz hatte, sagte das ab.**
c. ??**Ein Mann, der auch nur irgendeinen Rest von Stolz hatte, sagte das ab.**
d. ??**Manche Männer, die auch nur irgendeinen Rest von Stolz hatten, sagten das ab.**

Presuppositional Quantifier Phrases

So far, all quantifying determiners denote total functions from $D_{<e,t>}$ to $D_{<<e,t>,t>}$. The case seems to be different with presuppositional quantifying determiners, which might denote partial functions:

(31) a. **Neither cat has stripes**
b. **Both cats are in the kitchen**

Observation:
If, in (31-a), there are exactly two cats, and neither one has stripes, then $[\![(31\text{-a})]\!] = 1$; if there are exactly two cats, and at least one of them has stripes, then $[\![(31\text{-a})]\!] = 0$; and if there are not exactly two cats, then $[\![(31\text{-a})]\!]$ should be undefined because of a presupposition failure, as was the case with **the** (cf. the Fregean approach above). Similar conclusions apply in the case of (31-b). Thus:

(32) a. $[\![\textbf{neither}]\!] = [\ \lambda f \in D_{<e,t>}$ & there are exactly two x such that $f(x) = 1$. $[\ \lambda g \in D_{<e,t>}\ .\ \neg\exists y \in D_e\text{: } f(y) = 1\ \&\ g(y) = 1\ ]]$
b. $[\![\textbf{both}]\!] = [\ \lambda f \in D_{<e,t>}$ & there are exactly two x such that $f(x) = 1$. $[\ \lambda g \in D_{<e,t>}\ .\ \forall y \in D_e\text{: } f(y) = 1 \rightarrow g(y) = 1\ ]]$

Neither/No and Both/Every

Exercise, p. 154

$[\![$**neither**$]\!] = [\![$**no**$]\!]$ plus cardinality = 2 presupposition for the restriction.
$[\![$**both**$]\!] = [\![$**every**$]\!]$ plus cardinality = 2 presupposition for the restriction.

Formally:

(33) $[\![$**neither**$]\!] =$
$\lambda f \in D_{<e,t>}$ & there are exactly two x such that $f(x) = 1$. $[\![$**no**$]\!](f)$ $=$
$\lambda f \in D_{<e,t>}$ & there are exactly two x such that $f(x) = 1$. $[\ \lambda h \in D_{<e,t>}\ .\ [\ \lambda g \in D_{<e,t>}\ .\ \neg\exists y \in D_e$: $h(y) = 1\ \&\ g(y) = 1\ ]](f)$ $=$
$\lambda f \in D_{<e,t>}$ & there are exactly two x such that $f(x) = 1$. $[\ \lambda g \in D_{<e,t>}\ .\ \neg\exists y \in D_e$: $f(y) = 1\ \&\ g(y) = 1\ ]$

(34) $[\![$**both**$]\!] =$
$\lambda f \in D_{<e,t>}$ & there are exactly two x such that $f(x) = 1$. $[\![$**every**$]\!](f)$ $=$
$\lambda f \in D_{<e,t>}$ & there are exactly two x such that $f(x) = 1$. $[\ \lambda h \in D_{<e,t>}\ .\ [\ \lambda g \in D_{<e,t>}\ .\ \forall y \in D_e$: $h(y) = 1 \rightarrow g(y) = 1\ ]](f)$ $=$
$\lambda f \in D_{<e,t>}$ & there are exactly two x such that $f(x) = 1$. $[\ \lambda g \in D_{<e,t>}\ .\ \forall y \in D_e$: $f(y) = 1 \rightarrow g(y) = 1\ ]$

Other Examples for Presuppositional Quantifying Determiners

(35) a. **The two cats are in the kitchen**
$[\![$**the two**$]\!] = [\![$**both**$]\!]$
b. **The 25 cats are in the kitchen**
etc.

(36) Generalization on quantifying determiners of the **the n** type:
the n α has a semantic value only if there are exactly n x such that $[\![\alpha]\!](x) = 1$.
Where defined, $[\![\textbf{the n } \alpha]\!] = \lambda g \in D_{<e,t>} . \forall y \in D_e$: $[\![\alpha]\!](y) = 1 \rightarrow g(y) = 1$.

Exercise, p. 158

Suppose that $[\![\textbf{the cat}]\!] = [\![\textbf{the one cat}]\!]$. Then, it is clear that the denotation of the definite article **the** cannot be a function from $D_{<e,t>}$ to D_e anymore (i.e., it cannot pick out a specific, unique individual anymore), but must be a function from $D_{<e,t>}$ to $D_{<<e,t>,t>}$, just like other quantifying determiners. Still, uniqueness is ensured, and, of course, the analysis remains strictly presuppositional (since we are still dealing with a partial function):

(37) The Fregean presuppositional analysis of **the** adopted so far:
$[\![\textbf{the}]\!] =$
$\lambda f : f \in D_{<e,t>}\ \&\ \exists! x \in D_e\text{: } f(x) = 1\ .\ \iota y \in D_e\text{: } f(y) = 1$

(38) The new Fregean presuppositional analysis of **the**:
$[\![\textbf{the}]\!] =$
$[\![\textbf{the one}]\!] =$
$\lambda f \in D_{<e,t>}\ \&\ \exists! x \in D_e\text{: } f(x) = 1\ .\ [\ \lambda g \in D_{<e,t>}\ .\ \forall y \in D_e\text{: } f(y) = 1 \rightarrow g(y) = 1\ ]$

Frege vs. Russell

The denotation of **the** in (38) has the same logical type as other quantifying determiners, and, in particular, the same logical type as Russell's non-presuppositional, quantificational analysis of **the**:

(39) Russell's quantificational analysis of **the**:
$[\![\textbf{the}]\!] =$
$\lambda f \in D_{<e,t>} . [\lambda g \in D_{<e,t>} . \exists x \in D_e\text{: } \forall y \in D_e\text{: } [f(x) = 1 \leftrightarrow y = x] \,\&\, g(x) = 1]$

Clearly, Frege's and Russell's approaches to **the** differ with respect to presuppositions. E.g., $[\![$(40-a)$]\!] = 0$ under Russell's analysis, and undefined (presupposition failure) under either of Frege's analyses. Furthermore, $[\![$(40-b)$]\!] = 1$ under Russell's analysis (even though there is no escalator in South College), and again undefined under either of Frege's analyses.

(40) a. **The escalator in South College is dirty**
b. **John doesn't use the escalator in South College**

Both types of approaches, however, share the property that the denotation of **the** must be relativized to utterance contexts. (See above, **The door is locked**.)

Are All Determiners Presuppositional?

(41) Presupposition failure, not true sentences

a. **All/every American king(s) lived in New York**.
b. **All unicorns have accounts at the Chase Manhattan Bank**.

(42) Two lexical entries for universal quantifying determiners:

a. $[\![\textbf{every}]\!]_1 = [\ \lambda f \in D_{<e,t>}\ .\ [\ \lambda g \in D_{<e,t>}\ .\ \forall x \in D_e\text{: } f(x) = 1 \rightarrow g(x) = 1\ ]]$
b. $[\![\textbf{every}]\!]_2 = [\ \lambda f \in D_{<e,t>}$ & there is an x such that $f(x) = 1\ .\ [\ \lambda g \in D_{<e,t>}\ .\ \forall x \in D_e\text{: } f(x) = 1 \rightarrow g(x) = 1\ ]]$

(43) Oscillation beteween presupposition failure and truth (a) and falsity (b)

a. **No American king lived in New York**.
b. **Two American kings lived in New York**.

(44) Even more complicates cases – everyone accepts these sentences as true

a. **Every unicorn has exactly one horn**.
b. **Every unicorn is a unicorn**.