EXERCISES (STAT MECH II)

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Exercise I (SS 2009):

Problem 1:
Consider a system of N independent distinguishable linear oscillators with the potential energy

$$U_N(q_1, q_2 \cdots, q_i) = \sum_{i=1}^{N} \frac{m\omega^2 q_i^2}{2} \tag{1}$$

in the canonical ensemble.

Calculate the free energy $F(N, V, T)$ and the pressure $p(N, V, T)$ and the chemical potential $\mu(N, V, T)$ of the system:

a) in the semi-classical approximation

b) in quantum mechanics, where the energy levels ϵ_n of ONE harmonic oscillator are given by $\epsilon_n = \hbar\omega\left(\frac{1}{2} + n\right)$ with $n = 0, 1, 2 \ldots$).

c) Show that the following relation holds

$$N\mu(N, V, T) = F(N, V, T)$$

for both the semi-classical approximation and the quantum mechanical treatment.

Hints:
$\sum_{\nu=0}^{\infty} a^\nu = \frac{1}{1-a} \quad (0 < a < 1) \qquad \int_{-\infty}^{\infty} \mathrm{e}^{-x^2} \mathrm{d}x = \sqrt{\pi}$

Problem 2:
In an open (grand-canonical) μ, V, T-system the grand potential $\Phi(\mu, V, T)$ is related to the semi-classical partition function function $\mathcal{Z}(\mu, V, T)$ by

$$\Phi(\mu, V, T) = -kT \ln \mathcal{Z}(\mu, V, T) \tag{2}$$

and it holds

$$\frac{pV}{kT} = \mathcal{Z}(\mu, V, T) \tag{3}$$

a) Derive the relation (3).

b) Calculate $\frac{pV}{NkT}$ for an ideal gas using (3).

Hint: remember Euler's equation and $\Phi \equiv U - TS - \mu N$