EXERCISES (STAT MECH II)

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Exercise IX (SS 2009):

Problem 1:

For large clusters the cluster number distribution of a Bethe lattice reads

$$n_s(p) \propto s^{-5/2} \exp(-cs) \quad \text{with} \quad c \propto (p - p_c)^2 \quad (p \to p_c, \quad s \gg 1) \tag{1}$$

a.) Estimate the moments $M_k = \sum_s s^k n_s$ of (1)
b.)The correlation length ξ of the Bethe lattice divergers for $p \to p_c$ as

$$\xi \propto |p - p_c|^{-\nu} \qquad \text{(with critical exponent} \quad \nu = 1/2) \tag{2}$$

The fractal dimension D governing the cluster radius R_s ($s \propto R_s^D$) is related to ξ via a ratio of moments of (1) by

$$\xi = \left(\frac{M_{2+2/D}}{M_2} \right)^{1/2} \qquad (p \to p_c) \tag{3}$$

Estimate the fractal dimension D by comparing (2) and (3).

Problem 2:

Derive the relation between R_s and the average distance between two cluster sites

$$2R_s^2 = \sum_{i,j} \frac{|\vec{r}_i - \vec{r}_j|^2}{s^2} \tag{4}$$

The cluster radius is defined by

$$R_s^2 = \sum_{i=1}^{s} \frac{|\vec{r}_i - \vec{r}_0|^2}{s} \qquad \text{with} \qquad \vec{r}_0 = \sum_{i=1}^{s} \frac{\vec{r}_i}{s} \tag{5}$$

where $\vec{r}_i$ is the position vector of site i and $\vec{r}_0$ the position of the 'center of mass' of the cluster.