Morphologische Theorien
VII. Beschränkungen für Paradigmen 2

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Background

Goal:
Systematic accounts of as many instances of syncretism as possible: one form – one function (i.e., feature specification)

Consequence:
Abstractness of morphological analyses of paradigms:
- underspecification of exponents
- competition of exponents (and principles that resolve the competition)
- abstract features on exponents encoding natural classes (like [oblique], [local person])
- subanalysis of exponents: obvious in some cases, possible in many more cases (e.g., du, dich = /d/-/u:/, /d/-/ɪ/-/ç/; see Pike (1965), Wiese (2001), Fischer (2006)).

Question:
Is there independent evidence for such an approach?
Carstairs-McCarthy’s Constraints

(1) **The Paradigm Economy Principle** (Carstairs (1987, 51)):
When in a given language L more than one inflectional realization is available for some bundle or bundles of non-lexically-determined morphosyntactic properties associated with some part of speech N, the number of macroparadigms for N is no greater than the number of distinct “rival” macroinflections available for that bundle which is most generously endowed with such rival realizations.
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(2) **The No Blur Principle** (Carstairs-McCarthy (1994, 742)): Within any set of competing inflectional realizations for the same paradigmatic cell, no more than one can fail to identify inflection class unambiguously.
Observation:
(i) Constraints like the Paradigm Economy Principle and No Blur restrict the number of possible inflection classes that can be generated on the basis of a given set of inflection markers (for a given grammatical category).
(ii) If such constraints cannot be adopted for principled reasons, there is a danger that the theory is not restrictive enough.
(iii) Principled reasons that preclude adopting constraints on the number of possible inflection classes (on the basis of a given marker inventory):

- non-existence of paradigms in morphological theory
- decomposition of inflection class features in order to account for trans-paradigmatic syncretism (see Halle (1992), Oltra Massuet (1999), Stump (2001), Alexiadou & Müller (2007), and below).

(Compare Noyer’s (2005) Interclass Syncretism Constraint, which is similar in its effects to No Blur, and fundamentally incompatible with a decomposition of inflection class features.)
Strategies

Two possible strategies:

1. argue that the question of how inflection classes can be constrained is irrelevant from a synchronic perspective;

2. argue that restrictions on the number of possible inflection classes (based on a given marker inventory) follow from independently motivated assumptions, without invoking specific constraints that explicitly impose restrictions on possible inflection classes.

I adopt the latter strategy.
Inflection Class Economy Theorem

(3) **Inflection Class Economy Theorem:**
Given a set of \( n \) inflection markers, there can be at most \( 2^{n-1} \) inflection classes, independently of the number of instantiations of the grammatical category that the markers have to distribute over.

**Note:**
The number of \( 2^{n-1} \) inflection classes encodes the powerset of the inventory of markers, minus one radically underspecified marker.

For instance: Assuming an abstract system with five markers and six instantiations of a grammatical category (e.g., case), the Inflection Class Economy Theorem states that there can at most be sixteen (i.e., \( 2^{5-1} = 2^4 \)) inflection classes, out of the 15,625 (i.e., \( 5^6 \)) that would otherwise be possible.
Claim:
The Inflection Class Economy Theorem follows under any morphological theory that makes the three assumptions in (4), (5), and (6), which I call ‘Syncretism’, ‘Elsewhere’, and ‘Blocking’.

(I basically presuppose an approach along the lines of Distributed Morphology (Halle & Marantz (1993, 1994), Noyer (1992)), but things are exactly the same under alternative morphological theories, e.g., Minimalist Morphology (Wunderlich (1996, 1997)), or Paradigm Function Morphology (Stump (2001))).
(4) **Syncretism** (first assumption):
Identity of form implies identity of function: For each marker, there is a unique specification of morpho-syntactic features.
(within a certain domain, and unless there is evidence to the contrary).

**Note:**
The Syncretism Principle underlies much recent (and, based on the Jakobsonian tradition, some not so recent) work in inflectional morphology; it provides simple and elegant analyses, and it has been empirically confirmed for a variety of inflectional systems in the world’s languages.
Elsewhere

(5) **Elsewhere** (second assumption):
There is always one elsewhere marker that is radically underspecified with respect to inflection class (and more generally). Other markers may be underspecified to an arbitrary degree (including not at all).

**Note:**
(i) Underspecification as a means to account for syncretism is employed in most recent theories of inflectional morphology, including Distributed Morphology, Minimalist Morphology, and Paradigm Function Morphology.
(ii) The assumption that there is always one radically underspecified elsewhere marker in inflectional systems is quite common (see, e.g., Stump’s (2001) Identity Function Default rule).
(ii-a) It is well-motivated empirically because it can account for ‘discontinuous’ occurrences of markers in paradigms (where natural classes captured by non-radical underspecification is unlikely to be involved).
(ii-b) It ensures that there are (usually) no paradigmatic gaps in inflectional systems (which should otherwise be an option, given underspecification).
Blocking

(6) **Blocking** (third assumption):
Competition of underspecified markers is resolved by choosing the most specific marker: For all (competing) markers $\alpha$, $\beta$, either $\alpha$ is more specific than $\beta$, or $\beta$ is more specific than $\alpha$.

**Note:**
A Specificity constraint along these lines is adopted in Distributed Morphology (typically as part of the definition of the Subset Principle, see Halle (1997)), in Minimalist Morphology (see Wunderlich (1996, 1997, 2004)), and in Paradigm Function Morphology (Stump (2001) calls the relevant constraint Panini’s Principle).
Consequence:

1. Syncretism is systematic in the sense that only one specification of morpho-syntactic features is associated with any given inflection marker (with the qualifications mentioned above).

2. For any given fully specified context, there is always one inflection marker that fits.

3. For any given fully specified context, there is never more than one inflection marker that fits.

(Elsewhere and Blocking emerge as two sides of the same coin; see ‘Completeness’ and ‘Uniqueness’ in Wunderlich (1996, 99).)
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Two remaining issues:

- How does the Inflection Class Economy Theorem constrain inflectional systems?
- How does the Inflection Class Economy Theorem follow as a theorem from Syncretism, Elsewhere, and Blocking?
Basic Question

(7) Two versions of the basic question:

a. Given an inventory of markers for a certain domain (e.g., noun inflection), how many inflection classes can there be?

b. Given an inventory of markers with associated features encoding a grammatical category (e.g., case) for a certain domain (e.g., noun inflection), how many inflection classes can there be?

Assumption:
(7-a) is the more interesting question: It does not presuppose that the specification of a marker for a grammatical category (e.g., with respect to case and/or number) is somehow privileged, i.e., more basic than its inflection class features. (Carstairs (1987) only tries to answer (7-b).)
A system without restrictions

If, in a given domain (e.g., noun inflection), there are $n$ markers for $m$ instantiations of a grammatical category (e.g., case), the markers can be grouped into $n^m$ distinct inflection classes (i.e., the set of $m$-tuples over an input set with $n$ members).

**Abstract example 1**: 3 markers, 4 cases: $81 (= 3^4)$ possible inflection classes

<table>
<thead>
<tr>
<th>a a a a</th>
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</table>
Comments on Example 1

Note:
(i) The letters a, b, and c stand for the three markers.
(ii) All four-letter rows (4-tuples separated by either a vertical line or a line break) correspond to one inflection class, with the first marker in a row being used for the first instantiation of case (e.g., nominative), the second one for the second instantiation of case (e.g., accusative), the third one for the third instantiation of case (e.g., dative), and the fourth one for the fourth instantiation of case (e.g., genitive).
(iii) It is unlikely that a language can be found in which eighty-one inflection classes have been generated on the basis of three markers and four instantiations of a grammatical category.
Predictions for Example 1

(8)  a. Paradigm Economy Principle, worst case scenario:
     3 inflection classes: the size of the inventory
b. No Blur Principle, worst case scenario:
     9 inflection classes: \(((3-1)\times4)+1\)
c. Inflection Class Economy Theorem, worst case scenario:
     4 inflection classes: \(2^3-1\)
Explanation of worst case scenarios, Paradigm Economy Principle:
All three markers can be allomorphs for a single case specification (e.g., a, b, and c can all be accusative markers); still, there can then only be three distinct inflection classes.
Explanation of Worst Case Scenarios 2

(10) Explanation of worst case scenarios, No Blur Principle:

a. There is one default marker (say, \(a\)).

b. One class consists only of default markers (aaaa).

c. All the other inflection classes differ from this class by replacing one of the \(a\)'s with either \(b\) or \(c\) (baaa, abaa, aaba, aaab, caaa, acaa, aaca, aaac), so that all classes respect the No Blur Principle.

d. Adding another class with more than one \(b\), or more than one \(c\), or a – perhaps minimal – combination of \(b\)'s and \(c\)'s (cf. bbaa, or aacc, or abca, etc.) will invariably lead to a violation of the No Blur Principle because either \(b\) or \(c\) (or both) will cease to be inflection-class specific.

e. In general, the No Blur Principle predicts that there can at most be \(((n-1) \times m)+1\) inflection classes, for \(n\) markers and \(m\) instantiations of a grammatical category: Every marker except for one – the default marker, hence “–1” – can appear for a given instantiation of a grammatical category only in one inflection class; and “+1” captures a class consisting exclusively of default markers.
Note:
Assuming default markers that are specific with respect to instantiations of a grammatical category (such that, e.g., \(a\) is the default marker for the first instantiation, \(b\) for the second, \(c\) for the third, and perhaps again \(a\) for the fourth) instead of an extremely general default marker \(a\), does not change things: This would be compatible with No Blur, but it could not increase the number of possible inflection classes. In the case at hand, the maximal set of inflection classes would include \(abca, bbca, cbca, aaca, acca, abaa, abba, abcb, abc\).
Another Abstract Example

Abstract example 2: 5 markers, 3 cases: 125 (= 5^3) possible inflection classes

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Predictions for Example 2

(11) a. Paradigm Economy Principle, worst case scenario: 5 inflection classes: the size of the inventory
b. No Blur Principle, worst case scenario: 13 inflection classes: \(((5-1)\times 3)+1\)
   (E.g., assuming a as a default marker, aaa, baa, aba, aab, caa, aca, aac, daa, ada, aad, eaa, aea, aae)
c. Inflection Class Economy Theorem, worst case scenario: 16 inflection classes: \(2^{5−1}\)
Example 3: 5 markers, 4 cases: \(625 (= 5^4)\) inflection classes
Predictions for Example 3

(12)  

a. Paradigm Economy Principle, worst case scenario:  
   5 inflection classes: the size of the inventory  

b. No Blur Principle, worst case scenario:  
   17 inflection classes: \((5-1) \times 4 + 1\)  
   (E.g., aaaa, baaa, abaa, aaba, aaab, caaa, acaa, aaca, aaac, daaa, 
   adaa, aada, aaad, eaaa, eaa, aaea, aaee.)  

c. Inflection Class Economy Theorem, worst case scenario:  
   16 inflection classes: \(2^{5-1}\)

Conclusion so far:  
The Inflection Class Economy Theorem restricts possible inflection classes in a 
way that is roughly comparable to the Paradigm Economy and No Blur Principles.
Assumptions

Recall:

1. **Syncretism**: Only one morpho-syntactic feature specification is associated with each marker of the inventory for a given morphological domain (exceptions apart).

2. **Elsewhere**: There is always one marker that in principle fits into every context of fully specified morpho-syntactic features.

3. **Blocking**: There is always only one marker that can in fact be used for any fully specified context of morpho-syntactic features.
Argument Based On Marker Deactivation Combinations 1

(13) a. Since each inflection marker M can only be associated with one specification of morpho-syntactic features (because of Syncretism), it follows that for each inflection marker M and for each inflection class I, it must be the case that M is either compatible with I or incompatible with I.

b. A marker is compatible with an inflection class I if it bears no inflection class feature, if it bears fully specified inflection class information that completely characterizes I, or if it is characterized by a set of underspecified inflection class features that is a subset of the fully specified set of features that characterize the inflection class.

c. M is activated for I if it is compatible with it; and deactivated for I if it is incompatible with it.

(If a marker is activated for an inflection class I, this does not imply that it will actually be used by I – there may well be a more specific marker that blocks it.)
(13) a. Since each inflection marker M can only be associated with one specification of morpho-syntactic features (because of **Syncretism**), it follows that for each inflection marker M and for each inflection class I, it must be the case that M is either **compatible** with I or **incompatible** with I.

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(If a marker is activated for an inflection class I, this does not imply that it will actually be used by I – there may well be a more specific marker that blocks it.)

d. **Blocking** ensures that each inflection class can be defined in terms of the markers that are active in it: For all competing markers \(\alpha\) and \(\beta\), it is fixed once and for all by the markers’ feature specifications (and independently of inflection classes) that either \(\beta\) is more specific than \(\alpha\), or \(\alpha\) is more specific than \(\beta\).

e. Hence, if the same set of markers is activated for two inflection classes \(I_1\) and \(I_2\), \(I_1\) must be identical to \(I_2\).
(13) f. Conversely, since every marker is either activated or deactivated for any given inflection class, it also follows that if the same set of markers is deactivated for two inflection classes $I_1$ and $I_2$, $I_1$ and $I_2$ must be the same inflection class (because the same set of markers is then activated for $I_1$ and $I_2$, because a marker $/\times/$ can only have one specification $[\xi]$, and because specificity relations among competing markers are fixed).
Conversely, since every marker is either activated or deactivated for any given inflection class, it also follows that if the same set of markers is deactivated for two inflection classes $I_1$ and $I_2$, $I_1$ and $I_2$ must be the same inflection class (because the same set of markers is then activated for $I_1$ and $I_2$, because a marker /x/ can only have one specification [ξ], and because specificity relations among competing markers are fixed).

In order to determine the maximal number of inflection classes on the basis of a given inventory of markers, it now suffices to successively deactivate all possible marker combinations.

Starting with the full inventory of markers, we can proceed by successively deactivated all combinations of markers, which yields class after class.

Thus, all markers of the inventory are compatible with class $I_1$; all except for marker a are compatible with class $I_2$; all except for markers a, b are compatible with class $I_3$; and so forth.
Argument Based On Marker Deactivation Combinations 2

(13) f. Conversely, since every marker is either activated or deactivated for any given inflection class, it also follows that if the same set of markers is deactivated for two inflection classes $I_1$ and $I_2$, $I_1$ and $I_2$ must be the same inflection class (because the same set of markers is then activated for $I_1$ and $I_2$, because a marker $/x/$ can only have one specification $[\xi]$, and because specificity relations among competing markers are fixed).

g. In order to determine the maximal number of inflection classes on the basis of a given inventory of markers, it now suffices to successively deactivate all possible marker combinations.

h. Starting with the full inventory of markers, we can proceed by successively deactivating all combinations of markers, which yields class after class.

i. Thus, all markers of the inventory are compatible with class $I_1$; all except for marker $a$ are compatible with class $I_2$; all except for markers $a$, $b$ are compatible with class $I_3$; and so forth.

j. However, by assumption (Elsewhere), one marker always is the elsewhere (default) marker: It is compatible with all inflection classes because it is radically underspecified; and therefore it cannot be deactivated by definition.

k. Consequently, all possible marker deactivation combinations are provided by the powerset of the set of all the markers of the inventory minus the elsewhere marker: $2^{n-1}$, for $n$ markers.

l. Thus, given a set of $n$ inflection markers, there can be at most $2^{n-1}$ marker deactivation combinations.

m. Marker deactivation combinations fully determine possible inflection classes. Hence: Given a set of $n$ inflection markers, there can be at most $2^{n-1}$ inflection classes.
Inflection Class Economy as A Theorem

Note:
This reasoning is independent of the number of instantiations of the grammatical category (e.g., the number of cases) that a set of markers needs to distribute over. In contrast to what is the case under the No Blur Principle, an increase in instantiations of a grammatical category does not induce an increase in possible inflection classes over a given inventory of markers. Hence:

(14) Inflection Class Economy Theorem:
Given a set of \( n \) inflection markers, there can be at most \( 2^{n-1} \) inflection classes, independently of the number of grammatical categories that the markers have to distribute over.
The First Example

Note:
In order to illustrate the possible marker deactivation patterns, the case categories are now called 1, 2, 3, and 4. Given an inventory of three markers, there are $2^{3-1} = 4$ deactivation combinations.

(15) Example 1 revisited:
   a. 3 markers: \{a, b, c\}
   b. 4 cases: 1, 2, 3, 4
   c. Deactivation combinations: \{ \{b, c\}, \{b\}, \{c\}, \{\} \}
The First Example 2

Observation:
Of the 81 inflection classes that would logically be possible under, only four remain, given Syncretism, Underspecification, and Blocking (i.e., the Inflection Class Economy Theorem). This result holds under any specificity-induced order of the markers, and under any assignment of case features to markers.

(16) A possible assignment of case specifications to markers:

a. Markers:
   (i) /a/ ↔ [ ]
   (ii) /b/ ↔ [12]
   (iii) /c/ ↔ [234]

b. Specificity:
   /b/ > /c/ > /a/

c. Deactivation combinations and inflection classes:
   {b, c} → aaaa
   {b} → accc
   {c} → bbbaa
   { } → bbcc
The First Example 3

(17) Another possible assignment of case specifications to markers:

a. Markers:
   (i) /a/ ↔ [ ]
   (ii) /b/ ↔ [234]
   (iii) /c/ ↔ [4]

b. Specificity:
   /c/ > /b/ > /a/

c. Deactivation combinations and inflection classes:
   \{b, c\} → aaaa
   \{b\} → aaac
   \{c\} → abbb
   \{ \} → abbc

Note:
The question of how the cases 1, 2, 3, 4 are derived from more primitive decomposed features (e.g., how [234] can be a natural class), and how systems with apparently unnatural classes (under minimal decomposition) are derived, is orthogonal.
The Third Example

Example 3 revisited:

a. 5 markers: \{a, b, c, d, e\}
b. 4 cases: 1, 2, 3, 4
### The Third Example 2

**A possible choice:**

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<td>{ } → cbde</td>
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### Another possible choice:

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<td>{b, c, d, e} → aaaa</td>
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<td>{b, c, d} → aaee</td>
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<td>{b, c, e} → adaa</td>
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<td>{b, c} → adee</td>
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<td>{b, d, e} → caaa</td>
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<td>{b, e} → cdaa</td>
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<td>{b} → cdee</td>
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<td>{c, d, e} → bbbb</td>
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<td>{c, d} → bbee</td>
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<td>{c, e} → bdab</td>
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<td>{c} → bdee</td>
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<td>{d, e} → cbbb</td>
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<td>{d} → cbee</td>
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<td>{e} → cdab</td>
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<td>{ } → cdee</td>
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The Third Example 3

(21) A third possible choice:

a. Markers:
   (i) /a/ ↔ [ ]
   (ii) /b/ ↔ [234]
   (iii) /c/ ↔ [134]
   (iv) /d/ ↔ [123]
   (v) /e/ ↔ [123]

b. Specificity:
   /d/ > /e/ > /c/ > /b/ > /a/

c. Deactivation combinations
   & inflection classes:
   \{b, c, d, e\} → aaaa
   \{b, c, d\} → eeea
   \{b, c, e\} → ddda
   \{b, c\} → ddda
   \{b, d, e\} → cacc
   \{b, d\} → eeec
   \{b, e\} → dddc
   \{b\} → dddc
   \{c, d, e\} → abbb
   \{c, d\} → eeeb
   \{c, e\} → dddb
   \{c\} → dddb
   \{d, e\} → cbcc
   \{d\} → eeec
   \{e\} → dddc
   \{\} → dddc

(22) A fourth possible choice:

a. Markers:
   (i) /a/ ↔ [ ]
   (ii) /b/ ↔ [1]
   (iii) /c/ ↔ [2]
   (iv) /d/ ↔ [3]
   (v) /e/ ↔ [4]

b. Specificity:
   /e/ > /d/ > /c/ > /b/ > /a/

c. Deactivation combinations
   & inflection classes:
   \{b, c, d, e\} → aaaa
   \{b, c, d\} → aaee
   \{b, c, e\} → aade
   \{b, c\} → aade
   \{b, d, e\} → acaa
   \{b, d\} → acee
   \{b, e\} → acda
   \{b\} → acde
   \{c, d, e\} → baaa
   \{c, d\} → baae
   \{c, e\} → bada
   \{c\} → bade
   \{d, e\} → bcaa
   \{d\} → bcde
   \{e\} → bcda
   \{\} → bcde
The Third Example 4

Note:
Again, the issue of what the decomposed case and inflection class features that encode the deactivation patterns in systems like (19)–(22) would actually look like is strictly speaking orthogonal to present concerns. Still, for the case at hand, in the worst case there would have to be four binary inflection class features $[\pm\alpha]$, $[\pm\beta]$, $[\pm\gamma]$ and $[\pm\delta]$ whose cross-classification yields the sixteen inflection classes (with individual markers underspecified as, e.g., $[+\alpha]$); two abstract grammatical category features (e.g., case features such as $[\pm\text{governed}]$, $[\pm\text{oblique}]$, as in Bierwisch (1967)) would suffice for all systems but (21), where either reference to negated specifications would be necessary, or a third primitive feature would have to be invoked.
Abstractness of Inflection Markers

The notion of “marker” is to be understood in a somewhat more abstract way that ignores allomorphic variation which is phonologically or morpho-phonologically conditioned (and not morphologically, as with variation determined by inflection class membership). For instance, Halle (1994) argues that the marker realizations ov and ej for genitive plural in Russian are allomorphs whose choice is morpho-phonologically determined; on this view, there is but a single marker /ov/, accompanied a single underspecified set of morpho-syntactic features (perhaps involving underspecified inflection class features, as suggested in Alexiadou & Müller (2007) in order to account for fact that this marker exhibits trans-paradigmatic syncretism).
Scope of the Result

There may be minor imperfections in inflectional systems that can be traced back to historical factors. In particular, these deviations from optimal design show up in the form of isolated markers that cannot be given unique specifications, resulting in a case of non-systematic homophony. In such a situation, the set of possible inflection classes is mildly increased; it is $2^{n-1+x}$, for $x$ additional marker specifications required by unresolved, accidental homophony.
The same reasoning applies to

1. the use of disjunction or negation in marker specifications (see, e.g., Bierwisch (1967), Wunderlich (1996)), but only if contradictory feature specifications are involved;

On the Other Hand

The $2^{n-1}$ formula captures worst case scenarios. Overlapping marker specifications reduce the number of possible inflection classes further. Moreover, for an inflectional system to fully exploit the logical possibilities for developing inflection classes as they arise under the Inflection Class Economy Theorem is extremely unlikely – typically, far from all marker deactivation combinations will be employed.
Consequences for Other Morphological Operations

(23) a. **Fission** (Distributed Morphology; Halle & Marantz (1993), Noyer (1992)), **rule blocks** (stem-and-paradigm accounts; Anderson (1992), Stump (2001)). Both concepts give rise to instances of subanalysis, in the sense that what may look like a complex marker at first sight turns out to be best analyzed as a sequence of smaller markers, each with its own specifications (Janda & Joseph (1992), Bierkandt (2006)): unproblematic as long as it is understood that no more than one inflection class can determine a sequence of subanalyzed markers in each case.

b. **Impoverishment** (Distributed Morphology): Given that standard impoverishment (as feature deletion) can be reanalyzed as insertion of a highly specific null marker (Trommer (1999)), each impoverishment rule also increases the set of n’s (for which the powerset is created) by one.


Conclusion


