Molecular Physics (Prof. Käs) **WS 2007/08**
Universität Leipzig
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Problem Set 13

Due date: January 28, 2008

Problem 48)
Estimate the maximum laser power of a cylindric ruby crystal with 4.0 cm length and 0.6 cm diameter in a pulse of 120 ns duration ($\lambda = 694.3$ nm). The ruby consists of 0.050 Cr^{3+} mass percent within the Al_2O_3 matrix with an overall density of 3.97 g/cm^3. Assume that the pumping radiation is sufficient to pump all chromium ions out of the ground state at a rate faster than they decay back to the ground state.
(4 points)

Problem 49)
Evaluate the results of a rotation-vibration spectrum with two spectral branches (infrared) given in cm^{-1} :

J	0	1	2	3	4	5	6
$^1H^{35}Cl$ (R branch)	2906.25	2925.92	2944.99	2963.35	2981.05	2998.05	3014.50
$^1H^{35}Cl$ (P branch)		2865.14	2843.63	2821.59	2799.00	2775.77	2752.01

For the energy levels (in units of the wavenumber), the following relation holds:

$$S(v,J) = \left(v + \tfrac{1}{2}\right)\tilde{\nu} + B_v J(J+1)$$

The lines of the rotation-vibration spectrum appear at $\Delta S_J^{P,R,Q} = S(v+1,J') - S(v,J'')$. For the P branch holds $J'=J''-1$, for the R branch holds $J'=J''+1$, and for the Q branch hold $J'=J''$.
Derive an expression for $\Delta S_{J-1}^{R} - \Delta S_{J+1}^{P}$ in the general case and make then use the values given in the table in order to calculate a mean for value for B_v. Use the reduced mass $\mu(^1H^{35}Cl)=1{,}6266\cdot10^{-27}$ kg.
(7 points)

Problem 50)
To describe the intramolecular energy of a diatomic molecule the Morse potential is used in the form $V(x) = D_e \cdot (1 - e^{-a\cdot x})^2$, with $x = r - r_e$ and r_e , the equilibrium separation of the nuclei. D_e is the (spectroscopic) dissociation energy and the constant a is defined as $a = \sqrt{\frac{\mu_{red}}{2D_e}}\,\omega$.

The values for the H_2 molecule are: $r_e = 74.1$ pm, $D_e = 7{,}61 \times 10^{-19}$ J and $a = 0.0193$ (pm)$^{-1}$.
a) Display $V(x)$ for H_2 in the range -50 pm $< r - r_e <$ 50 pm graphically.
b) Approximate the Morse potential close to r_e by a parabola and use the latter to estimate the relevant force constant k, the classical vibration frequency (in cm^{-1}) and the zero-level energy of the quantized oscillator.
(7 points)