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# Problem Set

Due date: Nov 26

**Problem 25**
A microscopic particle with mass $m$ is confined to a 1-$D$ box with length $L$. Its normalized wavefunction is

$$\psi_n = \sqrt{\frac{2}{L}}\sin\left(\frac{n\pi x}{L}\right), \qquad n = 1,2,3,\ldots$$

(a) Calculate $\langle p\rangle$ and $\langle p^2\rangle$. Give an explanation of your results.

(b) Show explicitly that the wavefunctions are orthonormal.

(4 points)

**Problem 26**
A classical particle with mass $m$ traveling to the right with an energy $E$ that encounters a potential barrier with $V$ (> $E$) and width $L$ will be reflected. In quantum mechanics there is a finite probability to find the particle on the other side of the barrier continuing to travel to the right. Following the derivation sketched out in the lecture, show that the transmission probability $T$ can be described by the following formula:

$$T = \frac{|A'|^2}{|A|^2} = \left\{1 + \frac{\left(e^{\kappa L} - e^{-\kappa L}\right)^2}{16\varepsilon(1-\varepsilon)}\right\}^{-1}$$

Here, $\varepsilon = \frac{E}{V}$ and $\kappa = \frac{\sqrt{2m(V-E)}}{\hbar^2}$. $A$ and $A'$ are the amplitudes of the wavefunctions of the particle moving to the right in front and behind the barrier, respectively. Also show that for very high or wide potential barriers ($\kappa L >> 1$) the transmission probability approaches

$$T = 16\varepsilon(1-\varepsilon)e^{-2\kappa L}.$$

(5 points)

**Problem 27**
The normalized wavefunction of a 1-$D$ harmonic oscillator with mass $m$ in a potential $V = \frac{1}{2}kx^2$ is given by

$$\psi_v = \sqrt{\frac{1}{\alpha\sqrt{\pi}2^v v!}}H_v\left(\frac{x}{\alpha}\right)e^{-\frac{x^2}{2\alpha^2}},$$

where $H_v(y)$ are the Hermite polynomials and $\alpha = \left(\frac{\hbar^2}{mk}\right)^{\frac{1}{4}}$. Use the recursion relation for Hermite polynomials to calculate $\langle V\rangle$ and $\langle x^3\rangle$.

(4 points)