UNIVERSITÄT LEIPZIG

Experimental Physics IV IPSP

Problem Set 4

Deadline: Thursday, 05.05.2011, before the lecture

Problem 10: 4+3 points

Two observers move towards each other. One sends a signal to the receiver. Classically (signal = sound), the formula for the Doppler-shifted frequency is given by

$$f_D = \frac{c + v_D}{c - v_S} f_S$$

with the propagation speed of the signal c, the velocity of the detector v_D, the velocity of the source v_S, the emitted frequency of the source f_s and the detected frequency f_D.

Furthermore, in the equivalent case of a relativistic signal (e.g. light, EM wave) the time dilation, which describes the difference of elapsed time between two relative moving observers, is given by

$$\Delta T' = \frac{\Delta T}{\sqrt{1 - v^2/c^2}}$$

with the elapsed time $\Delta T'$ in the reference frame of the detector and the elapsed time ΔT in the reference frame of the source with the relative velocity v.

a) Show that the relativistic Doppler-shifted frequency can be calculated to

$$f_{D,\mathrm{rel}} = \frac{\sqrt{1 + v/c}}{\sqrt{1 - v/c}} f_S$$

depending only on the relative speed v. Compare it to the classical Doppler-effect. What is special?

b) How fast do you have to drive in order to observe a red ($\lambda_{\mathrm{red}} = 670\ \mathrm{nm}$) traffic light “switching” to green ($\lambda_{\mathrm{green}} = 547\ \mathrm{nm}$) due to the relativistic Doppler-effect?

Hint: Einstein’s postulate in *On the electrodynamics of Moving Bodies*: The speed of light is constant in all inertial systems, regardless how they move relative to each other. Therefore, Use an appropriate frame of reference.

Problem 11: 2+4 points

The wave function for the ground state of the hydrogen-atom is given by

$$\Psi(r,t) = \frac{1}{\sqrt{\pi}\, a_0^{3/2}}\, e^{-\frac{r}{a_0}}\, e^{i\frac{E}{\hbar}t}$$

with the Bohr radius a_0.

a) Calculate the radial probability distribution $P(r)$ using the formula

$$P(r)\mathrm{d}r = |\Psi(r,t)|^2 \mathrm{d}V$$

b) Calculate the most probable distance r_0 and the mean radius $\langle r \rangle$ given by

$$\langle r \rangle = \langle \Psi | r | \Psi \rangle = \int \Psi^* r \Psi \mathrm{d}V = \int_0^\infty r P \mathrm{d}r$$

Hint:

$$\mathrm{d}V = \mathrm{d}x\mathrm{d}y\mathrm{d}z = \mathrm{d}^3 r$$

$$\int_0^\infty x^n e^{-\alpha x} \mathrm{d}x = \frac{n!}{\alpha^{n+1}}$$

Problem 12: 7+1 points

The probability distribution of a particle in a one-dimensional box (dimension of the box: $0 < x < d$) is given by

$$P_{\text{classical}}(x) = \frac{1}{d} \, ,$$

$$P_{n,\text{quantum}}(x) = \frac{2}{d} \sin^2\left(\frac{n\pi x}{d}\right) \, , \quad n = 1,2,3, \dots$$

for a classical and quantum-mechanical particle, respectively.

a) Calculate the expectation values $\langle x \rangle$ and $\langle x^2 \rangle$ for the classical and the quantum-mechanical particle in a box.
b) What happens in the limit case of increasing n?

Hint:

$$\int \sin^2 x \, \mathrm{d}x = \frac{x}{2} - \frac{1}{4} \sin 2x \, ,$$

$$\int x \sin^2 x \, \mathrm{d}x = \frac{x^2}{4} - \frac{x}{4} \sin 2x - \frac{1}{8} \cos 2x \, ,$$

$$\int x^2 \sin^2 x \, \mathrm{d}x = \frac{x^3}{6} - \left(\frac{x^2}{4} - \frac{1}{8}\right) \sin 2x - \frac{x}{4} \cos 2x \, .$$