UNIVERSITÄT LEIPZIG

Experimental Physics IV IPSP

Problem Set 10

Deadline: Thursday, 20.06.2012, before the seminar

Problem 32: 5 points

Electrons that were accelerated with 5000 V are shot on a double slit. 10m behind the double slit is a screen where the diffraction pattern is recorded. What is the minimal distance between the two slits, when you want to detect all maxima with a standard light microscope with a numerical aperture of 0.5? The microscope uses light of the wavelength 550nm.

Hints: Use the small angle approximation for the calculation of the distance between two maxima or minima. The mass of the electron is m = 511 keV/c².

Problem 33: 2+4 points

The wave function for the ground state of the hydrogen-atom is given by

$$\Psi(r,t) = \frac{1}{\sqrt{\pi}\, a_0^{3/2}}\, e^{-\frac{r}{a_0}}\, e^{i\frac{E}{\hbar}t}$$

with the Bohr radius a_0.

a) Calculate the radial probability distribution $P(r)$ using the formula

$$P(r)\mathrm{d}r = |\Psi(r,t)|^2 \mathrm{d}V$$

b) Calculate the most probable distance r_0 and the mean radius $\langle r \rangle$ given by

$$\langle r \rangle = \langle \Psi | r | \Psi \rangle = \int \Psi^* r \Psi \mathrm{d}V = \int_0^\infty r P \mathrm{d}r$$

Hint:

$$\mathrm{d}V = \mathrm{d}x\mathrm{d}y\mathrm{d}z = \mathrm{d}^3 r$$

$$\int_0^\infty x^n e^{-\alpha x}\,\mathrm{d}x = \frac{n!}{\alpha^{n+1}}$$

Problem 34: 1+1+1+1+1 points

Let A, B and C be operators. The commutator is defined as

$$[A, B] = AB - BA\,.$$

Show the the following commutator relations:

a) $[A, B + C] = [A, B] + [A, C]$,
b) $[A, BC] = [A, B]C + B[A, C]$.

The angular momentum operator $L = (L_x, L_y, L_z)$ is defined as:

$$L = r \times p\,.$$

Calculate the following commutators:

c) $[x, L_x]$ and $[p_x, L_x]$,
d) $\left[L_x, L_y\right]$,
e) $[L_x, L^2]$. *Hint:* "c + d + f = g"