UNIVERSITÄT LEIPZIG

Experimental Physics IV IPSP

Problem Set 11

Deadline: Thursday, 27.06.2012, before the seminar

Problem 35: 2+1+2+1+2+2 points

The wave function of a particle in a box ($0 < x < d$) is given by

$$\Psi_\mathrm{n}(x) = \sqrt{2/d}\ \sin\left(\frac{\pi n}{d}x\right).$$

Calculate:

a) $\langle n|m\rangle$
b) $\langle n|\hat{x}|n\rangle$ and $\langle n|\hat{x}^2|n\rangle$
c) $\langle n|\hat{p}|n\rangle$ and $\langle n|\hat{p}^2|n\rangle$
d) $\Delta x\Delta p$ with $\Delta f = \sqrt{\langle n|\hat{f}^2|n\rangle - \langle n|\hat{f}|n\rangle^2}$
e) $\langle n|\hat{H}|n\rangle$
f) What do you have calculated in the previous examples?

With $\hat{x}$, $\hat{p}$ *and* $\hat{H}$ as the space, momentum or Hamilton operator respectively and $\langle n|\hat{f}|m\rangle = \int \overline{\Psi_\mathrm{n}(x)}\, \hat{f}\Psi_\mathrm{m}(x)$.

Problem 36: 2+3+2 points

An incoming wave (coming from $-\infty$) with finite positive energy E is scattered at a potential $V(x) = -g\delta(x)$ with the delta distribution $\delta(x)$. One part of the incoming wave is reflected and the remaining is transmitted. Therefore, the general solution is

$$\Psi = \begin{cases} e^{ikx} + re^{-ikx} & \text{for} \quad x < 0 \\ te^{ikx} & \text{for} \quad x > 0 \end{cases}$$

with the wave vector $k = \sqrt{2mE}/\hbar$, $E > 0$ and $1 + r = t$

Calculate the Reflection and Transmission coeeficient $R = |r|^2$ and $T = |t|^2$.

a) Use your knowledge about the delta-distribution to verify the equation for the boundary condition:

$$\partial_x \Psi(0^-) - \partial_x \Psi(0^+) = \frac{2mg}{\hbar^2}\Psi(0)$$

b) Calculate r and t using the boundary condition above.

c) Finally, calculate R and T. Draw a sketch of the energy-dependent Reflection and transmission coefficient $R(E)$ and $T(E)$.

Problem 37: 2+1+3 points

Given is the time-dependant schrödinger equation for a free particle. Calculate first the general solution for this problem (with an arbritary starting condition) using the Fouriertransformation and afterwards for the starting conditions $\Psi(x, t = 0) = f(x)\delta(x)$ with the deltadistribution $\delta(x)$.

a) Plug in the fouriertransformation into your schrödinger equation and calculate the fouriertransformed wavefunction.

b) Give a formula for the fouriertransformed of your starting condition

c) Repeat a) and b) using $\Psi(x, t = 0) = f(x)\delta(x)$.

Hints for c):

$$\int_{-\infty}^{\infty} \frac{\cos(x)}{2\sqrt{|x|}} \mathrm{d}x = 0$$

$$\int_{-\infty}^{\infty} \frac{\mathrm{i}\sin(x)}{2\sqrt{|x|}} \mathrm{d}x \approx 2.5i$$