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Quantum Field Theory of Many-Particle Systems - Problem Set 1

Winter Term 2011/12

Due Date: Wednesday, October 19, 4 p.m., tutorial session

Internet: You can download this problem set at http://www.uni-leipzig.de/ rosenow → Teaching.

1. Field operators *5 Punkte*

The operators $a_k^\dagger$ and a_k create or annihilate single particle states with momentum k, respectively. They obey the commutation relations $[a_k, a_{k'}]_\zeta = 0$, $[a_k, a_{k'}^\dagger]_\zeta = \delta_{k,k'}$ with $\zeta = 1$ for bosons and $\zeta = -1$ for fermions. The field operator $\Psi(x)$ is defined as the Fourier transform

$$\Psi(x) \;=\; \frac{1}{\sqrt{L}} \sum_k a_k \, e^{ikx} \quad .$$

Show that $\Psi(x)$ and its Hermitian conjugate $\Psi^\dagger(x)$ obey the commutation relations

$$[\Psi(x), \Psi^\dagger(y)]_\zeta \;=\; \delta(x-y) \quad .$$

2. Second Quantization with field operators *10 Punkte*

A many-particle state is described by the Hamiltonian

$$H \;=\; \sum_{j=1}^{N} -\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x_j^2} + V(x_j) \;+\; \sum_{i<j} V(x_i - x_j) \quad ,$$

and the wave function $\varphi_\alpha(x_1, x_2, ..., x_N)$ is an eigenstate of H with eigenvalue E_α. In second quantized notation, the wave function φ_α is described by a state vector

$$|\varphi_\alpha\rangle \;=\; \int dx_1 dx_2 ... dx_N \; \varphi_\alpha(x_1, x_2, ..., x_N) \; \Psi^\dagger(x_1)\Psi^\dagger(x_2)...\Psi^\dagger(x_N)|0\rangle$$

with commutation relations of Ψ, $\Psi^\dagger$ as in probelm 1 supplemented by the condition that the operator $\Psi(x)$ annihilates the vacuum state $\Psi(x)|0\rangle = 0$. In second quantization, the Hamiltonian reads

$$H_s \;=\; \int dx \Psi^\dagger(x) \left(-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x_j^2} + V(x_j) \right) \Psi(x) \;+\; \frac{1}{2}\int dxdy \Psi^\dagger(x)\Psi^\dagger(y)V(x-y)\Psi(y)\Psi(x) \quad .$$

Show that

$$H_s|\varphi_\alpha\rangle = E_\alpha|\varphi_\alpha\rangle \quad .$$