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Quantum Field Theory of Many-Particle Systems - Problem Set 5

Winter Term 2011/12

Internet: You can download this problem set at http://www.uni-leipzig.de/∼rosenow.

7. Lehmann representation

3+3+3+3 Punkte

Derive the Lehmann representation for the retarded correlation function

$$C^{+}_{\hat{X}_1\hat{X}_2}(t) \;=\; -i\Theta(t)\langle[\hat{X}_1(t),\hat{X}_2(0)]_{\zeta_X}\rangle$$

and for the time-ordered correlation function in imaginary time

$$C^{\tau}_{\hat{X}_1\hat{X}_2}(\tau) \;=\; -\langle\hat{T}_\tau\hat{X}_1(\tau)\hat{X}_2(0)\rangle \quad .$$

a) Express the expectation value in the definition of the two correlation functions as a trace over the statistical operator $e^{-\beta(\hat{H}-\mu\hat{N})}$ using exact eigenstates $\{|\Psi_n\rangle\}$ of the full Hamiltonian. Insert a resolution of unity between the operators $\hat{X}_1$ and $\hat{X}_2$ to express the time evolution of $\hat{X}_1$ in terms of eigenvalues K_n of $\hat{K}=\hat{H}-\mu\hat{N}$.

b) Calculate the Fourier transforms

$$C^{+}(\omega) \;=\; \int_{-\infty}^{\infty} dt\; e^{i\omega t-\eta|t|}\; C^{+}(t)$$

and

$$C^{\tau}(i\omega_n) \;=\; \int_0^{\beta} d\tau e^{i\omega_n\tau}\, C^{\tau}(\tau) \quad .$$

by using the representations derived in part a). The imaginary time correlation function is (anti-) periodic with period β according to $C^\tau(\tau+\beta)=\zeta_X C^\tau(\tau)$ and can be Fourier transformed with respect to bosonic/fermionic Matsubara frequencies.

8. Fourier Transform of Lorentzian

5 Punkte

Calculate the Fourier integral

$$\int_{-\infty}^{\infty}\frac{d\omega}{2\pi}\, e^{-i\omega t}\;\; \frac{\Gamma}{\omega^2+\Gamma^2} \quad .$$

Use contour integration over a semi-circle (close the semi-circle in the lower or upper half plane depending on the sign of t) in connection with the residue theorem. Assume $\Gamma > 0$.