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## Quantum Field Theory of Many-Particle Systems - Problem Set 6

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Winter Term 2011/12

**Internet:** You can download this problem set at <http://www.uni-leipzig.de/~rosenow>.

### 9. Polarization Propagator in one dimension

4+4+4+4 Punkte

The polarization propagator in one spatial dimension is defined as

$$\Pi(i\omega_n, q) = 2 \int \frac{dk}{2\pi} T \sum_{\epsilon_l} \frac{1}{i\epsilon_l + i\omega_n - \xi(k+q)} \frac{1}{i\epsilon_l - \xi(k)} ,$$

where  $\xi(k) = \hbar^2 k^2 / 2m - \mu$  is the kinetic energy with respect to the chemical potential,  $\omega_n$  is a bosonic Matsubara frequency, and the sum runs over fermionic Matsubara frequencies  $\epsilon_l$ .

- As a first step towards evaluating the frequency sum, decompose the product of Green functions in the definition of  $\Pi$  into partial fractions.
- To perform the frequency sum, use the identity

$$T \sum_{\epsilon_l} \frac{e^{i\epsilon_l \eta}}{i\epsilon_l - \xi} = n_F(\xi) ,$$

where  $n_F(\xi) = 1/(e^{\beta\xi} + 1)$  denotes the Fermi distribution function.

- In order to perform the remaining momentum integral

$$-2 \int \frac{dk}{2\pi} \frac{n_F[\xi(k+q)] - n_F[\xi(k)]}{i\omega_n - \xi(k+q) + \xi(k)} ,$$

take the limit of zero temperature and expand both numerator and denominator to first order in  $q$ . Use the relation  $\partial n_F(\xi)/\partial \xi = -\delta(\xi)$  valid at zero temperature.

- Evaluate the momentum integral by using the identity

$$\int dx \delta[f(x)] = \sum_{x_i} \frac{1}{|f'(x_i)|}$$

where the  $x_i$  are the solutions of the equation  $f(x) = 0$ .