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Quantum Field Theory of Many-Particle Systems - Problem Set 11

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Internet: You can download this problem set at http://www.uni-leipzig.de/∼rosenow.

15. Meissner effect

5+5+5 Punkte

In the high temperature (static) limit, the effective action of a vector potential in a superconductor is

$$S_{\rm eff}[\mathbf{A}] \;=\; \frac{\beta}{2}\int d^dr \mathbf{A}^\perp(\mathbf{r})\cdot\left(-\frac{1}{\mu_0}\nabla^2 \;+\; \frac{n_s}{m}\right)\mathbf{A}^\perp(\mathbf{r}) \quad ,$$

where in Fourier space the transverse component of $\mathbf{A}$ is defined by

$$\mathbf{A}^\perp(\mathbf{q}) \;=\; \mathbf{A}(\mathbf{q}) \;-\; \frac{\mathbf{q}(\mathbf{q}\cdot\mathbf{A}(\mathbf{q}))}{q^2} \quad .$$

In the above action, μ_0 is the vacuum permeability, n_s is the superfluid density, and m is the electron mass.

a) Show that the gradient term in the effective action is equivalent to the standard magnetic field energy

$$\frac{1}{2\mu_0}\int d^dr\; \mathbf{B}(\mathbf{r})^2 \quad ,$$

where $\mathbf{B}(\mathbf{r}) \;=\; \nabla\times\mathbf{A}(\mathbf{r})$.

b) Show that the equation of motion for $\mathbf{A}(\mathbf{r})$ becomes

$$\nabla^2\mathbf{A} \;=\; \frac{1}{\lambda^2}\;\mathbf{A} \quad .$$

Derive an expression for the length scale λ in terms of n_s. This equation implies that a weak magnetic field will penetrate into a superconductor only up to the length scale λ.

c) To get a feeling for the penetration of a magnetic field into a superconductor, consider a semi-infinite slab of superconductor which occupies all of space for $x > 0$. Assume that for $x < 0$, there is a magnetic field $\mathbf{B} = (0, 0, B_0)$ pointing along the z-axis. Solve the equation derived in b) to get the profile of magnetic field inside the superconductor for $x > 0$.