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Quantum Field Theory of Many-Particle Systems - Problem Set 11

Winter Term 2011/12

Internet: You can download this problem set at http://www.uni-leipzig.de/∼rosenow.

15. Aharonov Bohm effect

4+4+4+4 Punkte

Consider a particle of charge e going above or below a very long impenetrable cylinder, as shown in the figure. Inside the cylinder is a magnetic field parallel to the cylinder axis, taken to be normal to the plane of the figure. Outside the cylinder there is no magnetic field. So the particle paths above and below enclose a magnetic flux, although the particle is not subject to a magnetic field. Our goal is to study how the probability of finding the particle in the interference region depends on the magnetic flux enclosed by interference paths going above and below the cylinder.

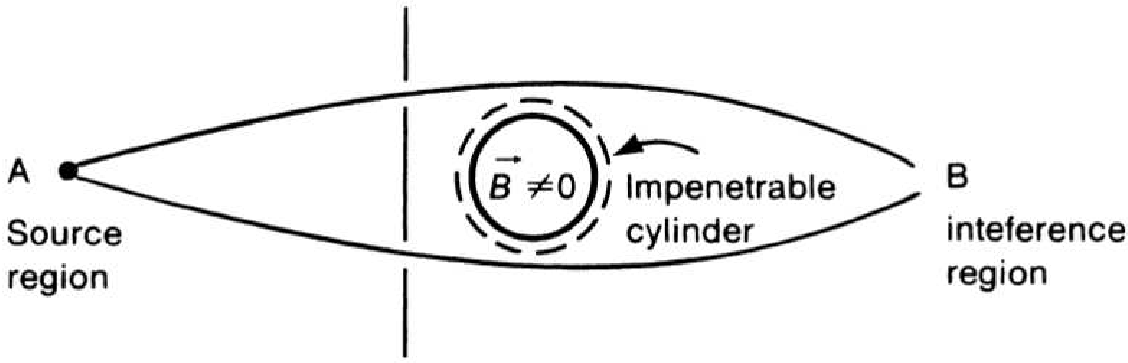

Abbildung 1: Setup for observation of the AB effect.

a) Inside the region with $r < a$, which is not accessible for the particle, the magnetic field is given by $B\mathbf{e}_z$, outside this region it vanishes. Verify that the magnetic field described above can be derived from the vector potential

$$\mathbf{A}(r,\varphi,z) = \begin{cases} \frac{1}{2}Br\ \mathbf{e}_\varphi, & r < a \\ (a^2B)/(2r)\ \mathbf{e}_\varphi, & r > a \end{cases}$$

b) The action for a charged particle moving in the presence of a vector potential is

$$S[x(t)] \ = \ \int_{t_i}^{t_f} dt \, \left(\frac{1}{2} m\dot{\mathbf{x}}^2 + e\,\dot{\mathbf{x}} \cdot \mathbf{A} \right) \quad .$$

Express the amplitude for the electron to travel from the source region to the interference region as a Feynman path integral. Convert the part of the action containing the vector potential into a line integral of type

$$\int_{x_i}^{x_f} \mathbf{A} \cdot d\mathbf{x} \quad .$$

Does such a line integral depend on the precise path chosen?

c) Split the sum over all paths in the Feynman path integral into sums over paths going above the cylinder and paths going below the cylinder. Assume that there are no paths winding around the cylinder multiple times. Show that after splitting the paths as described above, the contribution due to the vector potential can be factored out of the Feynman path integral for each class of paths, i.e.

$$\langle x_F, t_F | x_i, t_i \rangle \;=\; F_{\text{above}}[\mathbf{A}] \int_{\text{above}} D[x(t)] e^{\frac{i}{\hbar} S[x]} \;+\; F_{\text{below}}[\mathbf{A}] \int_{\text{below}} D[x(t)] e^{\frac{i}{\hbar} S[x]} \quad .$$

d) The probability for finding the particle in the interference region is the modulus squared of the amplitude described in c). This probability has an interference contribution between paths above and below the cylinder. Show that this interference term has an oscillatory dependence on the flux enclosed in the cylinder.