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Quantum Field Theory of Many-Particle Systems - Problem Set 13

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Internet: You can download this problem set at http://www.uni-leipzig.de/∼rosenow.

17. Flux quantization *6+6 Punkte*

In the high temperature (static) limit, the action of a long wave length excitation of the order parameter phase θ (we parametrize $\Delta(\mathbf{r}) = \Delta_0 e^{2i\theta(\mathbf{r})}$) in the presence of a vector potential $\mathbf{A}$ is

$$\frac{\beta}{2}\int d^d r \left[\frac{n_s}{m}\left(\hbar\nabla\theta - e_0\mathbf{A}\right)^2 \;+\; \frac{1}{\mu_0}\left(\nabla\times\mathbf{A}\right)^2\right] \quad .$$

Here, $-e_0$ and m are the electron charge and mass, n_S is the superfluid density, and μ_0 the vacuum permeability.

a) By minimizing the above action, derive the equations satisfied by Θ and A. Show that these equations are consistent with the identification of the gauge invariant (i.e. physical) current as

$$\mathbf{j} \;=\; \frac{e_0 n_S}{m}\left(\hbar\nabla\theta - e_0\mathbf{A}\right) \quad .$$

In terms of this current, your equations should be

$$\begin{aligned} \nabla\cdot\mathbf{j} &= 0 \\ \nabla\times(\nabla\times\mathbf{A}) &= \mu_0\,\mathbf{j} \end{aligned}$$

The first equation is the continuity equation expressing charge conservation, the second equation is Ampere's law.

b) Now we consider the properties of a vortex configuration in θ. We consider a cylindrical sample with a hole running through the center. Assume now that the phase winds around by $-\pi$ (such that the order parameter $\propto e^{2i\theta}$ stays single valued) on going once around a loop that encircles the hole, i.e.

$$\int_{\mathcal{C}} d\mathbf{l}\cdot\nabla\theta \;=\; \pi \quad ,$$

where the integral is taken along the loop $\mathcal{C}$. Due to the Meissner effect, the magnetic field will extend only a distance λ from the edge of the hole into the superconductor. Deep inside the superconductor, the current will be zero. Show that this implies that there is a mangetic flux $\frac{h}{2e_0}$ associated with this vortex.