Institut für Theoretische Physik
Universität Leipzig

Prof. Dr. B. Rosenow

Quantum Field Theory of Many-Particle Systems - Problem Set 4

Summer Term 2014

Internet: You can download this problem set at http://www.uni-leipzig.de/∼stp.

5. Fermion coherent states

3+3+3+3+3 Punkte

Fermion coherent states and their properties are needed for the derivation of fermionic functional integrals. Consider a fermionic coherent states $|\eta\rangle = e^{-\sum_i \eta_i \hat{a}_i^\dagger}|0\rangle$, $\langle\eta| = \langle 0|e^{-\hat{a}_i\overline{\eta}_i}$ and verify the following identities:

a)

$$\langle\eta|a_i^\dagger \ = \ \langle\eta|\overline{\eta}_i$$

b)

$$a_i^\dagger|\eta\rangle \ = \ -\partial_{\eta_i}|\eta\rangle$$

and

$$\langle\eta|a_i \ = \ \partial_{\overline{\eta}_i}\langle\eta|$$

c)

$$\langle\eta|\nu\rangle \ = \ e^{\sum_i \overline{\eta}_i\nu_i}$$

d)

$$\int d(\overline{\eta},\eta)\ e^{-\sum_i \overline{\eta}_i\eta_i}\ \eta\rangle\langle\eta| \ = \ 1_F$$

Hint: proceed in analogy to the proof for bosonic states in class, i.e. show that the integral commutes with all operators in Fock space.

e)

$$\langle n|\eta\rangle\langle\eta|n\rangle \ = \ \langle\zeta\eta|n\rangle\langle n|\eta\rangle$$

Here, $|n\rangle$ is an n-particle state in Fock space.

6. Green functions in momentum space

5+5 Bonus Punkte

The time ordered Green function in momentum space is defined by the ground state expectation value

$$G(t,k) \ = \ -i\langle|\hat{T}_t\hat{a}(t,k)\hat{a}^\dagger(0,k)\rangle \ \ .$$

Here, we consider noninteracting particles with a dispersion relation $\epsilon(k)$ and chemical potential μ, and $\hat{a}(t,k)$, $\hat{a}^\dagger(t,k)$ are annihilation and creation operators in the Heisenberg picture. $\hat{T}_t$ denotes the time ordering operator. Evaluate the Green function for a) noninteracting bosons and b) noninteracting fermions at zero temperature. The ground state for bosons is the vacuum (i.e. $\mu < 0$), for fermions the Fermi see (Fermi creation and annihilation operators are defined with respect to the Fermi sea).