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Quantum Field Theory of Many-Particle Systems - Problem Set 5

Summer Term 2014

Internet: You can download this problem set at http://www.uni-leipzig.de/∼stp.

7. Lehmann representation   *3+3+3+3 Punkte*

Derive the Lehmann representation for the retarded correlation function

$$C^{+}_{\hat{X}_1 \hat{X}_2}(t) \;=\; -i\Theta(t)\langle [\hat{X}_1(t), \hat{X}_2(0)]_{\zeta_X}\rangle$$

and for the time-ordered correlation function in imaginary time

$$C^{\tau}_{\hat{X}_1 \hat{X}_2}(\tau) \;=\; -\langle \hat{T}_{\tau}\hat{X}_1(\tau)\hat{X}_2(0)\rangle \;\; .$$

a) Express the expectation value in the definition of the two correlation functions as a trace over the statistical operator $e^{-\beta(\hat{H}-\mu\hat{N})}$ using exact eigenstates $\{|\Psi_n\rangle\}$ of the full Hamiltonian. Insert a resolution of unity between the operators $\hat{X}_1$ and $\hat{X}_2$ to express the time evolution of $\hat{X}_1$ in terms of eigenvalues K_n of $\hat{K} = \hat{H} - \mu\hat{N}$.

b) Calculate the Fourier transforms

$$C^{+}(\omega) \;=\; \int_{-\infty}^{\infty} dt \; e^{i\omega t - \eta|t|} \; C^{+}(t)$$

and

$$C^{\tau}(i\omega_n) \;=\; \int_{0}^{\beta} d\tau e^{i\omega_n \tau} \, C^{\tau}(\tau) \;\; .$$

by using the representations derived in part a). The imaginary time correlation function is (anti-) periodic with period β according to $C^{\tau}(\tau + \beta) = \zeta_X C^{\tau}(\tau)$ and can be Fourier transformed with respect to bosonic/fermionic Matsubara frequencies.

8. Fourier Transform of Lorentzian   *5 Punkte*

Calculate the Fourier integral

$$\int_{-\infty}^{\infty} \frac{d\omega}{2\pi} \, e^{-i\omega t} \; \frac{\Gamma}{\omega^2 + \Gamma^2} \;\; .$$

Use contour integration over a semi-circle (close the semi-circle in the lower or upper half plane depending on the sign of t) in connection with the residue theorem. Assume $\Gamma > 0$.