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Quantum Field Theory of Many-Particle Systems - Problem Set 6

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Internet: You can download this problem set at http://www.uni-leipzig.de/∼rosenow.

9. Polarization Propagator in one dimension

4+4+4+4 Punkte

The polarization propagator in one spatial dimension is defined as

$$\Pi(i\omega_n, q) \; = \; 2\int \frac{dk}{2\pi} T \sum_{\epsilon_l} \frac{1}{i\epsilon_l + i\omega_n - \xi(k+q)} \frac{1}{i\epsilon_l - \xi(k)} \quad ,$$

where $\xi(k) = \hbar^2 k^2/2m \; - \; \mu$ is the kinetic energy with respect to the chemical potential, ω_n is a bosonic Matsubara frequency, and the sum runs over fermionic Matsubara frequencies ϵ_l.

a) As a first step towards evaluating the frequency sum, decompose the product of Green functions in the definition of Π into partial fractions.

b) To perform the frequency sum, use the identity

$$T \sum_{\epsilon_l} \frac{e^{i\epsilon_l \eta}}{i\epsilon_l - \xi} \; = \; n_F(\xi) \quad ,$$

where $n_F(\xi) = 1/(e^{\beta\xi} + 1)$ denotes the Fermi distribution function.

c) In order to perform the remaining momentum integral

$$-2\int \frac{dk}{2\pi} \frac{n_F[\xi(k+q)] - n_F[\xi(k)]}{i\omega_n - \xi(k+q) + \xi(k)} \quad ,$$

take the limit of zero temperature and expand both numerator and denominator to first order in q. Use the relation $\partial n_F(\xi)/\partial\xi = -\delta(\xi)$ valid at zero temperature.

d) Evaluate the momentum integral by using the identity

$$\int dx \; \delta[f(x)] \; = \; \sum_{x_i} \frac{1}{|f'(x_i)|}$$

where the x_i are the solutions of the equation $f(x) = 0$.