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Quantum Field Theory of Many-Particle Systems - Problem Set 10

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Internet: You can download this problem set at http://www.uni-leipzig.de/∼rosenow.

14. Tunneling between metal and superconductor *4+4+4+4 Punkte*

The most important verifications of the BCS theory of superconductivity came from electron tunneling experiments, in which the energy gap as a function of temperature was measured and showed excellent agreement with the BCS theory. In this problem electron tunneling between normal metal and superconductor will be explored. The notation is the same as in problem 13. There, we showed that the single particle tunnel current can be expressed as

$$\begin{aligned} I_{\text{single}} &= ie\left[C^{+}_{A,A^\dagger}(-eV) - \left(C^{+}_{A,A^\dagger}(-eV)\right)^{\star}\right] \\ &= -2e\,\mathrm{Im}[C^{+}_{A,A^\dagger}(-eV)] \quad . \end{aligned}$$

Here, the Fourier transform

$$C^{+}_{A,A^\dagger}(\omega) = \int_{-\infty}^{\infty} dt\, e^{i\omega t} C^{+}_{A,A^\dagger}(t)$$

of the retarded correlation function

$$C^{+}_{A,A^\dagger}(t) = -i\Theta(t)\langle[A(t), A^\dagger(0)]\rangle$$

is used, and the operator A is defined as

$$A = \sum_{\mathbf{k},\mathbf{p}} T_{\mathbf{k},\mathbf{p}} c^\dagger_{\mathbf{k}} c_{\mathbf{p}} \quad .$$

a) We want to obtain the retarded correlation function $C^{+}_{A,A^\dagger}(\omega)$ via analytic continuation from the imaginary time correlation function $C^{\tau}_{A,A^\dagger}(i\omega_n)$.

Show that

$$C^{\tau}_{A,A^\dagger}(i\omega_n) = \sum_{\mathbf{k},\mathbf{p}} |T_{\mathbf{k},\mathbf{p}}|^2\, T \sum_{i\epsilon_l} G_L(i\epsilon_l, \xi_{\mathbf{k}}) G_R(i\epsilon_l - i\omega_n, \xi_{\mathbf{p}}) \quad .$$

b) Use spectral representations

$$G_{L,R}(i\epsilon_l, \xi_{\mathbf{k},\mathbf{p}}) = \int \frac{d\omega}{2\pi} \frac{A(\omega, \xi_{\mathbf{k},\mathbf{p}})}{i\epsilon_l - \omega}$$

for both Green functions to evaluate the Matsubara sum in the expression for $C^{\tau}_{A,A^\dagger}(i\omega_n)$. You will have to perform a partial fraction decomposition and to make use of the identity

$$T\sum_{\epsilon_l}\frac{e^{i\eta\epsilon_l}}{i\epsilon_l-\xi_{\mathbf{k}}} \;=\; n_F(\xi_{\mathbf{k}}) \quad .$$

Analytically continue $i\omega_n \to \omega + i\eta$ to the retarded correlation function, and show that the current is given by

$$I \;=\; 2e\sum_{\mathbf{k},\mathbf{p}}|T_{\mathbf{k},\mathbf{p}}|^2\int_{-\infty}^{\infty}\frac{d\epsilon}{2\pi}A_R(\epsilon,\xi_{\mathbf{k}})A_L(\epsilon+eV,\xi_{\mathbf{p}})\left[n_F(\epsilon)-n_F(\epsilon+eV)\right] \quad .$$

c) Now make use of the fact that the the tunneling matrix elements can be approximated to be independent of momenta, i.e.

$$T_{\mathbf{k},\mathbf{p}} \;\equiv\; T_0 \quad ,$$

introduce the tunneling densities of states

$$\rho_L(\epsilon) \;=\; \frac{1}{2\pi}\sum_{\mathbf{k}}A_L(\epsilon,\xi_{\mathbf{k}}) \quad , \quad \rho_R(\epsilon) \;=\; \frac{1}{2\pi}\sum_{\mathbf{p}}A_R(\epsilon,\xi_{\mathbf{p}})$$

to show that the tunneling current is given by

$$I_{rmsingle} \;=\; 4\pi e\Omega_L\Omega_R|T_0|^2\int d\epsilon\,\rho_R(\epsilon)\rho_L(\epsilon+eV)\left[n_F(\epsilon)-n_F(\epsilon+eV)\right] \quad .$$

Here, Ω_L and Ω_R are the volume of the left and the right system, respectively.

d) Use now that for the normal metal $\rho_L(\epsilon)\equiv\rho_F$ and that for the supercondcuctor

$$\rho_R(\epsilon) \;=\; \rho_F\frac{|\epsilon|}{\sqrt{\epsilon^2-|\Delta_0|^2}}\,\Theta(\epsilon^2-\Delta^2)$$

to evaluate the tunnel current.