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Advanced Quantum Mechanics - Problem Set 4

Winter Term 2019/20

Due Date: Hand in solutions to problems marked with * before the lecture on **Friday, 15.11.2019, 09:15**. Due to the holiday on Wednesday, the problem set will only be discussed in the tutorial on Friday, 22.11.2019, 13:30-15:00, R. 210, ITP.

*10. Spin 1 system

3+2 Points

The Hamiltonian for a spin 1 system is given by

$$\hat{H} = A\hat{S}_z^2 + B(\hat{S}_x^2 - \hat{S}_y^2),$$

where the $\hat{S}_i$ are spin operators and A, B are real constants.

(a) Find the normalized energy eigenstates and eigenvalues.

(b) Is the Hamiltonian invariant under time reversal? How do the normalized eigenstates you calculated in part (a) transform under time reversal?

*11. Time reversal of a lattice Hamiltonian

2+3+2 Points

In this problem we will consider the effects of time reversal on a lattice Hamiltonian.

(a) First consider the lattice translation operator $\hat{T}_a = e^{-i\hat{p}a}$. How do the eigenvalues of the translation operator transform under time reversal?

(b) Now consider the Hamiltonian

$$H(\boldsymbol{k}) = \sin(k_x)\sigma_x + \sin(k_y)\sigma_y + M\sigma_z,$$

where k_x and k_y are components of the momentum appearing in the eigenvalues of the translation operator and M is a constant. How does this Hamiltonian transform in the case where σ are (i) spin matrices and (ii) some "orbital" matrices (such as in the problem on the SSH model)?

(c) Generalize your result to a Hamiltonian of the form $H(\boldsymbol{k}) = \boldsymbol{d}(\boldsymbol{k}) \cdot \boldsymbol{\sigma}$.