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Advanced Quantum Mechanics - Problem Set 3

Winter Term 2022/23

Due Date: Hand in solutions to problems marked with * to mailbox 39 with label "Advanced Quantum Mechanics" inside ITP room 105b before the lecture on **Friday, 04.11.2022, 09:15**. The problem set will be discussed in the tutorials on Monday 07.11.2022 and Wednesday 09.11.2022.

1. Translation Operator

1+2 Points

Consider a free particle with Hamiltonian

$$\hat{H} = \frac{\hat{p}^2}{2m},$$

and define the translation operator $\hat{T}_l$.

(a) Show that $[\hat{H}, \hat{T}_l] = 0$.

(b) Due to the result in (a), the Hamiltonian and translation operator have a common set of eigenstates. For such a state $|k\rangle$, calculate the eigenvalue of $\hat{T}_l$. That is calculate λ_k in the expression $\hat{T}_l|k\rangle = \lambda_k|k\rangle$.

2. Landau levels

3+3+2 Points

A spinless particle of charge q is confined to the x-y plane and subjected to a magnetic field in the z-direction, $\boldsymbol{B} = (0, 0, B)$.

(a) Using the Landau gauge $\boldsymbol{A} = (0, Bx, 0)$ show that the Schrödinger equation can be written as

$$\frac{-\hbar^2}{2m}\left(\frac{\partial^2}{\partial x^2} + \left(\frac{\partial}{\partial y} - i\frac{qB}{\hbar}x\right)^2\right)\Psi(x, y) = E\Psi(x, y).$$

Hint: You can use the minimal coupling rule to obtain the canonical momentum.

(b) Show that a solution of the Schrödinger equation above can be written as $\Psi(x, y) = e^{iky}u(x - a)$, and find an expression for a in terms of k. What does $u(x - a)$ look like? Explain why the energy eigenvalues are given by

$$E = \frac{\hbar qB}{m}\left(n + \frac{1}{2}\right), \qquad n = 0, 1, 2, \ldots$$

(c) The particles are now confined to an area of length X in the x-direction and Y in the y-direction. Using periodic boundary conditions, $\Psi(y) = \Psi(y + Y)$ in the y-direction, calculate the maximum value of n per unit area.

Hint: Don't forget that $a \leq X$.

*3. SSH model

4+2+3 Points

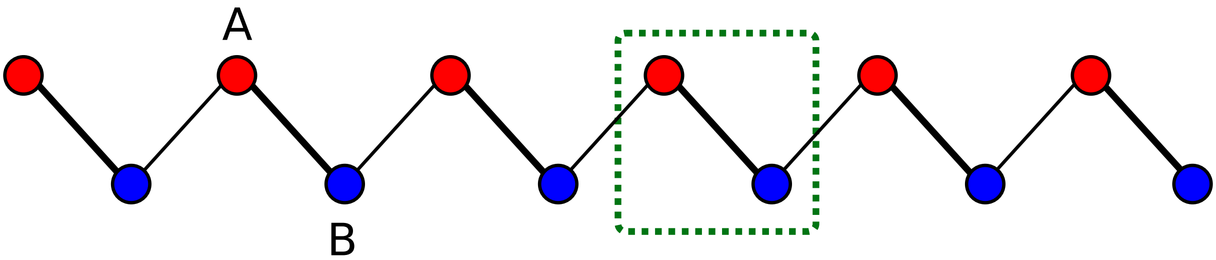

Figure 1: The SSH model. The red and blue circles symbolise different types of sites. The thin lines denote couplings with strength $t(1-\delta)$ whilst the thick lines are couplings with strength $t(1+\delta)$. The dashed square denotes a unit cell.

In this problem we consider the Su-Schrieffer-Heeger (SSH) model which describes spinless fermions hopping on a one-dimensional lattice with staggered hopping amplitudes (see the figure). The model contains two sub-lattices, A and B and has the following Hamiltonian

$$H = \sum_n t(1+\delta)|n, A\rangle\langle n, B| + t(1-\delta)|n+1, A\rangle\langle n, B| + \text{h.c.}.$$

Here h.c. stands for hermitian conjugate and $|n, A\rangle$ describes a state of site n, in sublattice A. t and δ are taken to be real parameters.

(a) By Fourier transforming, $|n\rangle = \frac{1}{\sqrt{N}} \sum_k e^{-ink} |k\rangle$, show that the Hamiltonian can be written as $H(k) = \boldsymbol{d}(k) \cdot \boldsymbol{\sigma}$, where $\boldsymbol{\sigma}$ is the vector of Pauli matrices, and $d_x(k) = t(1+\delta) + t(1-\delta)\cos(k)$, $d_y(k) = t(1-\delta)\sin(k)$, and $d_z(k) = 0$.

Hint: Write the wave function as a vector with two components describing the amplitudes on the A and B sublattices, respectively.

(b) Calculate the energy eigenvalues of the system.

(c) Plot your result from (b) for $\delta > 0$ and $\delta < 0$. What happens when $\delta = 0$?