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Advanced Quantum Mechanics - Problem Set 6

Winter Term 2023/24

Due Date: Hand in solutions to problems marked with * as a single pdf file using Moodle before the lecture on **Thursday, 23.11.2023, 15:15**. The problem set will be discussed in the tutorials on Monday 27.11.2023 and Wednesday 29.11.2023.

Website: https://home.uni-leipzig.de/stp/Quantum_Mechanics_2_WS2324.html

Moodle: https://moodle2.uni-leipzig.de/course/view.php?id=45746

1. Momentum-space wave functions *2 Points*

Let $\phi(\boldsymbol{p})$ be the momentum-space wave function for a state $|\alpha\rangle$, such that $\phi(\boldsymbol{p}) = \langle \boldsymbol{p}|\alpha\rangle$. Let also $\hat{\Theta}$ denote the time-reversal operator. Is the momentum-space wave function for the time-reversed state $\hat{\Theta}|\alpha\rangle$ given by $\phi(\boldsymbol{p})$, $\phi(-\boldsymbol{p})$, $\phi^*(\boldsymbol{p})$, or $\phi^*(-\boldsymbol{p})$? Justify your answer.

2. Graphene *3+1+3 Points*

The Hamiltonian for graphene near the $\boldsymbol{K}'$ point is given by

$$H = \hbar v_F \begin{pmatrix} 0 & q_x + iq_y \\ q_x - iq_y & 0 \end{pmatrix},$$

where v_F is the Fermi velocity.

(a) Calculate the normalized eigenstates of this Hamiltonian.

(b) We now consider next-nearest neighbors. The Hamiltonian is then modified by

$$H_{\text{nnn}} = -\frac{t'}{2} \sum_{\langle\langle i,j \rangle\rangle} (|i,A\rangle\langle j,A| + |i,B\rangle\langle j,B| + \text{h.c}),$$

where A and B denote different sub-lattices and the sum is over next-nearest neighbors. Write down the next-nearest neighbor lattice vectors.

(c) Show that the next-nearest neighbors give rise to an extra contribution to the spectrum of $-t'f(\boldsymbol{q})$ with

$$f(\boldsymbol{q}) = 2\cos\left(\sqrt{3}q_y a\right) + 4\cos\left(\frac{\sqrt{3}}{2}q_y a\right)\cos\left(\frac{3}{2}q_x a\right).$$

*3. Relativistic Landau Levels

3+3+2 Points

The low-energy Hamiltonian for electrons in graphene is given by

$$H = v_F \begin{pmatrix} -\boldsymbol{\sigma}^* \cdot \boldsymbol{p} & 0 \\ 0 & \boldsymbol{\sigma} \cdot \boldsymbol{p} \end{pmatrix},$$

where v_F is the Fermi velocity, $\boldsymbol{p}$ the momentum and $\boldsymbol{\sigma}$ the vector of Pauli matrices. The eigenstates can be written as four-dimensional state vectors with contributions from the $\boldsymbol{K}$ and $\boldsymbol{K}'$ points. That is, we write

$$\boldsymbol{\chi} = \begin{pmatrix} \chi'_A \\ \chi'_B \\ \chi_A \\ \chi_B \end{pmatrix}.$$

(a) Show that the eigenvalue equations decouple into

$$E^2 \chi_A = v_F^2 (p_x - i p_y)(p_x + i p_y) \chi_A,$$
$$E^2 \chi_B = v_F^2 (p_x + i p_y)(p_x - i p_y) \chi_B,$$

and similar for the primed parts of the eigenstates.

(b) Suppose now a magnetic field is switched on. Using the Landau gauge $\boldsymbol{A} = (-By, 0)$, perform the minimal substitution $\boldsymbol{p} \to \boldsymbol{p} - \frac{e}{c}\boldsymbol{A}$ in the eigenvalue equations in part (a) and deduce the form of the eigenfunctions.

(c) What does the energy spectrum look like?