Institut für Theoretische Physik
Universität Leipzig

Prof. Dr. B. Rosenow
N. John, M. Thamm

Advanced Statistical Physics - Problem Set 8

Summer Term 2018

Due Date: Tuesday, June 5, 09:15 a.m., mailbox inside ITP

Internet: Advanced Statistical Physics exercises

10. Correlation function I

2+2+1 Points

Consider a time series $\{s_1, s_2, s_3, ...\}$, where at each moment of time i the variable s_i can take values ± 1. At each time step Δt the variable changes its sign ($s_{i+1} = -s_i$) with probability p and keeps it value ($s_{i+1} = s_i$) with probability $1 - p$.

a) Show that the correlation function is given by $G(j - i) = \langle s_i s_j \rangle = (1 - 2p)^{|j-i|}$.

b) Denote $j - i = t/\Delta t$ and $\tau = \Delta t/(2p)$, and calculate the continuum limit $G(t)$ of the correlation function by assuming that τ is constant, but $\Delta t \to 0$. (Notice, that this means that $p \to 0$ i.e. we assume that the probability of the sign change decreases when we decrease the time step in our time series.)

c) Calculate the Fourier transform $G(\omega)$ of a correlation function $G(t) = e^{-|t|/\tau}$.

11. Correlation function II

3+3+3 Points

Consider the Ginzburg-Landau functional

$$\mathcal{H} = \int d^d x \left[\frac{a\tau}{2} \psi(\mathbf{x})^2 + \frac{c}{2} (\nabla \psi(\mathbf{x}))^2 - h(\mathbf{x})\psi(\mathbf{x}) \right] .$$

The associated Euler-Lagrange equation is given by

$$c\nabla^2 \psi(\mathbf{x}) = a\tau\psi(\mathbf{x}) - h(\mathbf{x}) .$$

a) Use the Fourier transformation to write down the formal solution of this equation for $h(\mathbf{x}) = h\delta^{(d)}(\mathbf{x})$. In the lectures it will be shown that this solution is equivalent with a two point correlation function.

b) Solve the Euler-Lagrange equation for $\tau = 0$ and $h(\mathbf{x}) = h\delta^{(d)}(\mathbf{x})$.
Hint: Use Gauss's theorem.

c) Solve the Euler-Lagrange equation for $\tau > 0$.
Hint: Assume that the solution is spherically symmetric and decays exponentially at large distances

$$\psi(\mathbf{x}) \propto \frac{e^{-r/\xi}}{r^p}.$$

Solve the equation in the limits $r \ll \xi$ and $r \gg \xi$.