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Advanced Statistical Physics - Problem Set 12

Summer Term 2018

Due Date: Tuesday, July 03, 09:15 a.m., mailbox inside ITP

Internet: Advanced Statistical Physics exercises

18. The differential recursion relations *2+2+2+2 Points*

The renormalization group procedure defines a mapping of the Hamiltonian with given parameters S into rescaled Hamiltonian with parameters S'. The rescaled parameters S' depend on the original parameters S and the rescaling factor $b = e^l$.
For the $d = 1+\epsilon$ dimensional Ising model, the differential recursion relations for the temperature T and the magnetic field h are

$$\begin{aligned} \frac{dT}{dl} &= -\epsilon T + \frac{T^2}{2} \\ \frac{dh}{dl} &= (1+\epsilon)h \ . \end{aligned}$$

a) Sketch the renormalization group flows in the (T, h) plane (for $\epsilon > 0$), marking the fixed points along the $h = 0$ axis.

b) Calculate the eigenvalues y_t and y_h, at the critical fixed point, to order of ϵ.

c) Starting from the relation governing the change of the correlation length ξ under renormalization, show that

$$\xi(t, h) = |t|^{-\nu} g_\xi(h/|t|^\Delta),$$

(where $t = T/T_c - 1$), and find the exponents ν and Δ.

d) Use a hyperscaling relation to find the singular part of the free energy $f_{\text{sing}}(t, h)$, and hence the heat capacity exponent α.

19. Coupled scalars *1+3+2+2+2 Points*

Consider the Hamiltonian

$$\beta\mathcal{H} = \int d^d x \left[\frac{t}{2} m^2 + \frac{K}{2} (\nabla m)^2 - hm + \frac{L}{2} (\nabla^2 \phi)^2 + v(\nabla m)(\nabla \phi) \right] ,$$

coupling two one-component fields m and ϕ.

a) Write $\beta\mathcal{H}$ in terms of the Fourier transforms $m(\boldsymbol{q})$ and $\phi(\boldsymbol{q})$.

b) Construct a renormalization group transformation by rescaling distances such that $\boldsymbol{q}' = b\boldsymbol{q}$, and the fields such that $m'(\boldsymbol{q}') = \tilde{m}(\boldsymbol{q})/z$ and $\phi'(\boldsymbol{q}') = \tilde{\phi}(\boldsymbol{q})/y$. You do not need to evaluate the integrals that just contribute a constant additive term.

c) There is a fixed point such that $K' = K$ and $L' = L$. Find y_t, y_h and y_v at this fixed point.

d) The singular part of the free energy has a scaling form

$$f(t, h, v) = t^{2-\alpha} g(h/t^{\Delta}, v/t^{\omega})$$

for t, h, v close to zero. Find α, Δ and ω.

e) There is another fixed point such that $t' = t$ and $L' = L$. What are the relevant operators at this fixed point, and how do they scale?