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Advanced Statistical Physics - Problem Set 13

Summer Term 2018

Due Date: Tueday, July 10, 09:15 a.m., mailbox inside ITP

Internet: Advanced Statistical Physics exercises

This exercise sheet is **not mandatory**, but you can solve it to get additional points. In case that you already have the 50 % of the points from the exercises, it will not be marked.

20. Higgs mechanism

3+3+4+4 Points

This problem is related to the Higgs mechanism. You will find that the coupling of the gauge field to the superconducting order parameter will give rise to a mass for the photons. The manifestation of the Higgs mechanism in superconductors is the Meissner effect.

The Landau-Ginzburg model of superconductivity describes a complex superconducting order parameter $\psi(\boldsymbol{x})$, and the electromagnetic vector potential $\boldsymbol{A}(\boldsymbol{x})$, which are subject to a Hamiltonian

$$\beta\mathcal{H} = \int d^d x \left[\frac{K}{2} D_\mu \psi D_\mu^* \psi^* + \frac{t}{2}|\psi|^2 + u|\psi|^4 + \frac{L}{2}(\nabla \times \boldsymbol{A})^2 \right] .$$

Here the gauge-invariant derivative $D_\mu = \partial_\mu - ieA_\mu(\boldsymbol{x})$ introduces a coupling between the two fields.

a) Show that the above Hamiltonian is invariant under the local gauge symmetry:

$$\psi(\boldsymbol{x}) \to \psi(\boldsymbol{x}) e^{i\theta(\boldsymbol{x})} , \qquad A_\mu \to A_\mu + \frac{1}{e}\partial_\mu \theta .$$

b) Show that there is a saddle point solution of the form $\psi(\boldsymbol{x}) = \overline{\psi}$ and $\boldsymbol{A} = 0$, and find $\overline{\psi}$ for $t > 0$ and $t < 0$.

c) For $t < 0$, calculate the cost of fluctuations by setting

$$\psi(\boldsymbol{x}) = (\overline{\psi} + \phi(\boldsymbol{x}))e^{i\theta(\boldsymbol{x})} , \qquad A_\mu(\boldsymbol{x}) = a_\mu(\boldsymbol{x}) .$$

and expanding $\beta\mathcal{H}$ to quadratic order in ϕ, θ and a_μ. Use the Coulomb gauge $\partial_\mu a_\mu = 0$. Further, you may assume that $\overline{\psi}, \phi(\boldsymbol{x}), a_\mu(\boldsymbol{x}) \in \mathbb{R}$.

d) Perform a Fourier transformation, and calculate the expectation values of $\langle|\phi(\boldsymbol{q})|^2\rangle$, $\langle|\theta(\boldsymbol{q})|^2\rangle$, and $\langle|\boldsymbol{a}(\boldsymbol{q})|^2\rangle$.