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Advanced Statistical Physics - Problem Set 8

Summer Term 2019

Due Date: Wednesday, June 05, 12:00 p.m., Hand in tasks marked with * to mailbox with label 'Advanced Statistical Physics Exercises' inside ITP room 105b

Internet: Advanced Statistical Physics exercises

10. Tricritical point*

4+2+2+4 Points

By tuning an additional parameter, a second order transition can be made first order. The special point separating the two types of transitions is known as a tricritical point, and can be studied by examining the Landau-Ginzburg Hamiltonian

$$\beta\mathcal{H} = \int d^d x \left[\frac{t}{2}m^2 + um^4 + vm^6 - hm\right] = \int d^d x \; \Psi(m),$$

where u can be positive or negative. For $u < 0$, a positive v is necessary to ensure stability.

a) By sketching the energy density $\Psi(m)$, for various t, show that in the saddle point approximation there is a first-order transition for $u < 0$ and $h = 0$.

b) Calculate the critical value of the parameter $t = \bar{t}(u)$ for this transition and the discontinuity in the magnetization $\overline{m}(u)$.

c) For $h = 0$ and $v > 0$, plot the phase boundary in the (u, t) plane, identifying the phases, and order of the phase transitions.

d) The special point $u = t = 0$, separating first and second order phase boundaries, is a tricritical point. For $u = 0$, calculate the tricritical exponents α, β, δ and γ, governing the singularities in heat capacity, magnetization and susceptibility. (Recall: $C \propto t^{-\alpha}$, $\overline{m}(h = 0) \propto t^{\beta}$, $\overline{m}(t = 0) \propto h^{1/\delta}$ and $\chi \propto t^{-\gamma}$.)

11. Correlation function I

2+2+1 Points

Consider a time series $\{s_1, s_2, s_3, ...\}$, where at each moment of time i the variable s_i can take values ± 1. At each time step Δt the variable changes its sign ($s_{i+1} = -s_i$) with probability p and keeps it value ($s_{i+1} = s_i$) with probability $1 - p$.

a) Show that the correlation function is given by $G(j - i) = \langle s_i s_j \rangle = (1 - 2p)^{|j-i|}$.

b) Denote $j - i = t/\Delta t$ and $\tau = \Delta t/(2p)$, and calculate the continuum limit $G(t)$ of the correlation function by assuming that τ is constant, but $\Delta t \to 0$. (Notice, that this means that $p \to 0$ i.e. we assume that the probability of the sign change decreases when we decrease the time step in our time series.)

c) Calculate the Fourier transform $G(\omega)$ of a correlation function $G(t) = e^{-|t|/\tau}$.