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Advanced Statistical Physics - Problem Set 2

Summer Term 2020

Due Date: Tuesday, April 21, 10:00 a.m., solutions must be mailed to stp2.leipziguni@gmail.com

1. Fourier transform

2 + 3 Points

a) This problem is a mathematical problem, which will help you to get familiar with Fourier transformations. The Fourier representation of a real field $\psi(x)$ is defined as

$$\psi(x) = \frac{1}{\sqrt{L^d}} \sum_{\mathbf{k}} e^{i\mathbf{k}\cdot\mathbf{x}} \psi_{\mathbf{k}}.$$

Show that the inverse transformation is given by

$$\psi_{\mathbf{k}} = \frac{1}{\sqrt{L^d}} \int d^d x \; e^{-i\mathbf{k}\cdot\mathbf{x}} \psi(x) = \alpha_{\mathbf{k}} + i\beta_{\mathbf{k}},$$

and further deduce that $\alpha_{\mathbf{k}} = \alpha_{-\mathbf{k}}$ and $\beta_{\mathbf{k}} = -\beta_{-\mathbf{k}}$.

b) Explicitly derive the Fourier transform of the Landau-Ginzburg Hamiltonian

$$\mathcal{H} = \int d^d x \left[\frac{a\tau}{2} \psi^2(x) + \frac{u}{4} \psi^4(x) + \frac{c}{2} (\nabla \psi)^2 - h(x)\psi(x) \right].$$

2. Tricritical point

4+2+2+4 Points

By tuning an additional parameter, a second order transition can be made first order. The special point separating the two types of transitions is known as a tricritical point, and can be studied by examining the Landau-Ginzburg Hamiltonian

$$\beta\mathcal{H} = \int d^d x \left[\frac{t}{2} m^2 + u m^4 + v m^6 - h m \right] = \int d^d x \; \Psi(m),$$

where u can be positive or negative. For $u < 0$, a positive v is necessary to ensure stability.

a) By sketching the energy density $\Psi(m)$, for various t, show that in the saddle point approximation there is a first-order transition for $u < 0$ and $h = 0$.

b) Calculate the critical value of the parameter $t = \bar{t}(u)$ for this transition and the discontinuity in the magnetization $\overline{m}(u)$.

c) For $h = 0$ and $v > 0$, plot the phase boundary in the (u, t) plane, identifying the phases, and order of the phase transitions.

d) The special point $u = t = 0$, separating first and second order phase boundaries, is a tricritical point. For $u = 0$, calculate the tricritical exponents α, β, δ and γ, governing the singularities in heat capacity, magnetization and susceptibility. (Recall: $C \propto t^{-\alpha}$, $\overline{m}(h = 0) \propto t^{\beta}$, $\overline{m}(t = 0) \propto h^{1/\delta}$ and $\chi \propto t^{-\gamma}$.)