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Advanced Statistical Physics - Problem Set 4

Summer Term 2020

Due Date: Tuesday, May 5, 10:00 a.m., solutions must be mailed to stp2.leipziguni@gmail.com

1. Functional derivatives *3 Points*

Derive the Euler-Lagrange equation corresponding to the following potential

$$F = \int_a^b \mathrm{d}x \, \left[\frac{c}{2}(\partial_x \psi)^2 + \frac{a\tau}{2}\psi^2(x) + \frac{d}{2}(\partial_x^2 \psi)^2 \right] ,$$

with the boundary values $\psi(a) = \psi_a$, $\psi(b) = \psi_b$, $\partial_x \psi(x)\big|_{x=a} = \psi'_a$, and $\partial_x \psi(x)\big|_{x=b} = \psi'_b$. You may follow the route of the usual variational calculus that you should have encountered in earlier courses. Therefore, assume that the stationary solution is $\psi(x)$ and consider the field $\psi_\lambda(x) = \psi(x) + \lambda\varepsilon(x)$, where $\varepsilon(x)$ is a deviation with the boundary conditions $\varepsilon(a) = \varepsilon(b) = \partial_x \varepsilon(x)\big|_{x=a} = \partial_x \varepsilon(x)\big|_{x=b} = 0$.

2. Correlation function II *3+3+3 Points*

Consider the Ginzburg-Landau functional

$$\mathcal{H} = \int d^d x \left[\frac{a\tau}{2}\psi(\mathbf{x})^2 + \frac{c}{2}(\nabla\psi(\mathbf{x}))^2 - h(\mathbf{x})\psi(\mathbf{x}) \right] .$$

The associated Euler-Lagrange equation is given by

$$c\nabla^2\psi(\mathbf{x}) = a\tau\psi(\mathbf{x}) - h(\mathbf{x}) .$$

a) Use the Fourier transformation to write down the formal solution of this equation for $h(\mathbf{x}) = h\delta^{(d)}(\mathbf{x})$. In the lectures it will be shown that this solution is equivalent with a two point correlation function.

b) Solve the Euler-Lagrange equation for $\tau = 0$ and $h(\mathbf{x}) = h\delta^{(d)}(\mathbf{x})$.
Hint: Use Gauss's theorem.

c) Solve the Euler-Lagrange equation for $\tau > 0$.
Hint: Assume that the solution is spherically symmetric and decays exponentially at large distances

$$\psi(\mathbf{x}) \propto \frac{e^{-r/\xi}}{r^p}.$$

Solve the equation in the limits $r \ll \xi$ and $r \gg \xi$.