Prof. Alexander Tsirlin

Problem sheet 10: Elastic and optical properties of metals **03.01.2024**

10.1. Compressibility of metals (6 P)

Delocalized (itinerant) electrons contribute to the bulk modulus of metals. Here, we employ the model of Fermi gas (free electron gas) to estimate this contribution.

(a) Use energy of the Fermi gas, $E = \frac{3}{5}N_e\varepsilon_F$, to derive its bulk modulus.

(b) Estimate electronic contribution to the bulk moduli of K and Ag.

10.2. Sum rule (7 P)

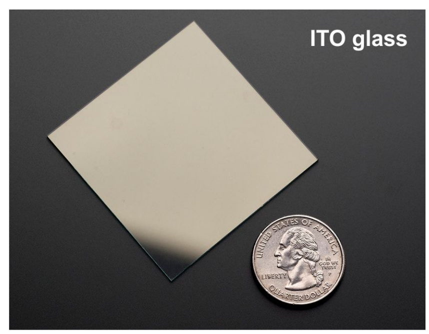
(a) Sketch real and imaginary parts of the optical conductivity for a Drude metal. Which power law, $1/\omega^\alpha$, is expected at high frequencies, $\omega \to \infty$?

(b) Determine the peak position and peak height in $\sigma''(\omega)$ for silver with $\rho = 1.7\,\mu\Omega\cdot$cm at room temperature.

(c) Calculate spectral weight, i.e., the area under the $\sigma'(\omega)$ curve of a Drude metal. Show that it does not depend on τ and should thus remain constant at different temperatures. This is the so-called *sum rule*.

(d) Calculate spectral weight for silver.

10.3. Which metal is better? (7 P)



Compare the performance of gold and indium-doped tin oxide (ITO) as heat-reflecting materials for windows. The electron concentration in ITO is $n_e = 1.0 \times 10^{21}\,\text{cm}^{-3}$. In the case of gold, estimate n_e according to its valence.

(a) Calculate plasma frequencies of both gold and ITO.

(b) Calculate reflectivities of gold and ITO for thermal radiation ($\lambda = 10\,\mu$m) and visible light ($\lambda = 500\,$nm). Use the simple ansatz $\varepsilon = 1 - \omega_p^2/\omega^2$ ($\tau \to \infty$), as shown in lecture 20.

(c) In real metals, reflection of light is accompanied by absorption. The absorption coefficient K enters Beer's law, $I = I_0\,e^{-Kz}$ (z is metal thickness), and depends on the imaginary part of the refractive index as $K = 4\pi n''/\lambda$. Use ε to determine n'' for the visible light ($\lambda = 500\,$nm) and estimate which fraction of this light should be absorbed by the 30 nm-thick gold foil. Compare to the result for ITO.