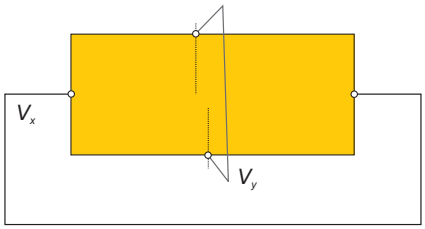
Problem sheet 11: Electron dynamics, Fermi surface **14.01.2024**

11.1. Hall effect measurements (6 P)



You measure Hall effect on a slab of gold with the dimensions of $2 \times 0.5 \times 0.2\,\mathrm{cm}^3$ ($x \times y \times z$). Room-temperature resistivity of gold is $\rho = 2.3\,\mu\Omega{\cdot}\mathrm{cm}$.

(a) Determine the Hall coefficient of gold and the Hall resistivity in the applied field of $B_z = 2\,\mathrm{T}$. Use electron concentration n_e calculated from the number of valence electrons per atom.

(b) The voltage $V_x = 2\,\mathrm{mV}$ is applied along the x-direction. Determine the Hall voltage along y.

(c) Experimentally, you measure the Hall voltage of $V_y = 6\,\mu\mathrm{V}$. Compare it to the value from (b). The discrepancy may be due to the different electron concentration n_e or due to an offset between the two contacts where V_y is measured (this offset will add a fraction of V_x to the value you measure). Using $n_e = 5.90 \times 10^{22}\,\mathrm{cm}^{-3}$, determine the offset between the contacts.

In real lab measurements, one uses five contacts (i.e., two contacts along one side of the sample) in order to compensate for such an offset.

11.2. Dynamics of electrons in parabolic bands (6 P)

Parameters of electron dynamics can be directly related to the band dispersion and band width. Consider a square lattice of atoms with the band energy $\varepsilon(\mathbf{q}) = \varepsilon_0 + 2t\cos(q_x a) + 2t\cos(q_y a)$, where a is the lattice constant and $t = \mathrm{const}$ is the hopping parameter (interaction between the adjacent atoms).

(a) Determine the band width.

(b) Show that band dispersion is parabolic when $|\mathbf{q}|a \ll 1$.

(c) Use the parabolic dispersion to determine electron velocity and density of states at $|\mathbf{q}|a \ll 1$.

(d) Determine the effective mass in the same $|\mathbf{q}|a \ll 1$ limit.

11.3. Back to the roots (4 P)

Apply general expression for the conductivity derived in lecture 24,

$$\sigma = \frac{e^2\tau}{4\pi^3\hbar} \cdot \int\limits_{\mathrm{FS}} \frac{[\mathbf{v}(\mathbf{q}) \otimes \mathbf{v}(\mathbf{q})]}{|\mathbf{v}(\mathbf{q})|}\, dS,$$

to the free electron gas. Show that conductivity of the Drude metal is recovered.

11.4. Shift of the Fermi surface (4 P)

Room-temperature resistivity of aluminum is $2.7\,\mu\Omega{\cdot}\mathrm{cm}$.

(a) Determine the mean-free time τ.

(b) Determine the shift of the Fermi surface, $\delta\mathbf{q}$, in the electric field of $100\,\mathrm{V/m}$. Compare this shift to the length of the Fermi vector, q_F.