Problem sheet 6: Phonons **22.11.2023**

6.1. Yet another dispersion relation (8 P)

(a) Obtain phonon dispersion relation for a diatomic chain with the alternating spring constants k_1 and k_2. Assume the same atomic mass m for all atoms.

(b) Explore the limits of $q \to 0$ and $q = \pi/a$.

(c) Explore the limits of $k_1 = k_2$ and $k_1 \gg k_2$. Explain the nature of the displacement waves in each of these two cases.

6.2. Earthquake assessment (7 P)

Olivine, $(Mg,Fe)_2SiO_4$, is the most abundant mineral in the Earth's mantle. Its bulk modulus can be approximated by $B = B_0 + B_0' p$ where $B_0 = 129\,\mathrm{GPa}$ is the bulk modulus at ambient pressure, and $B_0' = 4.3$. Its shear modulus $G = G_0 + g_1 p + g_2 p^2$ where $G_0 = 78\,\mathrm{GPa}$, $g_1 = 1.71$, and $g_2 = -0.027\,\mathrm{GPa}^{-1}$.

(a) Estimate the velocities of the p-wave (compressional wave) and s-wave (shear wave) at ambient pressure, assuming that olivine contains 50% of Mg and 50% of Fe.

(b) Derive pressure dependence of volume, $V(p)$.

(c) Estimate the p-wave and s-wave velocities at the depth of 200 km below the Earth's surface where pressure is about 10 GPa.

(d) An earthquake caused the s-wave signal arriving 40 s after the signal from the p-wave. Determine at what depth the earthquake took place, assuming linear dependence of both velocities on the depth, and assuming that the waves propagate perpendicular to the surface.

6.3. Ionic crystals from the phonon perspective (5 P)

(a) Use the experimental phonon frequencies to determine effective charges in the crystals of LiF ($\nu_{TO} = 9.6\,\mathrm{THz}$, $\nu_{LO} = 20.1\,\mathrm{THz}$) and GaP ($\nu_{TO} = 11.0\,\mathrm{THz}$, $\nu_{LO} = 11.9\,\mathrm{THz}$). Which crystal is more ionic?

(b) Determine the static permittivity assuming $\epsilon_\infty = 1$.