Problem sheet 8: Thermal expansion and thermal conductivity **06.12.2023**

8.1. Phonons and thermal expansion (7 P)

Unlike heat capacity, thermal expansion can not be described within the harmonic approximation. In modeling thermal expansion, anharmonicity is included in a rather unsophisticated way, via volume dependence of phonon frequencies. Here we assess the magnitude of this effect, namely, how much phonon frequencies depend on the volume.

(a) Use experimental data (e.g., from Crystallography Open Database) to determine linear (α_L) and volume (α_V) thermal expansion coefficients of NaCl in the temperature range of $100 - 300\,^\circ$C. The α values will change a bit with temperature, you can take the value averaged over this temperature interval.

(b) Assume that the Dulong-Petit limit for the heat capacity C_V is reached in this temperature range. Calculate the Grüneisen parameter Γ using the ambient-pressure bulk modulus of $B = 24.4\,$GPa.

(c) At ambient pressure, the average frequency of the optical phonon in NaCl is $\nu = 5.62\,$THz. Use the simplified form of $\Gamma = -(V/\omega)(d\omega/dV)$ to estimate how this frequency changes at 1 GPa. Compare to the relative change in volume at the same pressure. Assume that B is constant within this pressure range.

8.2. Heat losses (4 P)

You own a house, which is $10 \times 15\,\mathrm{m}^2$ in cross section and 4 m high.

(a) Calculate the heat loss through house walls that are 15 cm thick and that have an average thermal conductivity of $0.07\,\mathrm{W/m\,K}$ (twice that of glass wool). The inside and outside temperatures are $20\,^\circ$C and $0\,^\circ$C, respectively. Ignore the roof, doors, and windows.

(b) How many 1 kW room heaters would be needed to balance the heat transfer due to conduction?

(c) Your friend living in Norway boasts a better thermal insulation that reduces average thermal conductivity to $0.04\,\mathrm{W/m\,K}$ and adds 2 cm to the thickness of the walls. Installing this insulation costs 10 EUR per m^2. In what time will these costs pay off for your house if the cold season lasts for 4 months with the average ΔT of $15\,^\circ$C, the heating is electrical, and the electricity price is 0.40 EUR per kWh?

8.3. Phonon mean-free path (6 P)

Thermal conductivity of NaCl is $105.0\,\mathrm{W/m\,K}$ (20 K), $22.7\,\mathrm{W/m\,K}$ (100 K), and $6.0\,\mathrm{W/m\,K}$ (300 K).

(a) Determine heat capacity of NaCl at 20 K, 100 K, and 300 K using Debye's formula with the Debye temperature $\theta_D = 320\,$K.

(b) Brillouin light scattering experiment on NaCl performed with the green laser ($\lambda = 520\,$nm) in the 90° geometry shows resonances at 7.08 GHz and 13.7 GHz. Determine velocities of the longitudinal and transverse acoustic phonons near $q = 0$.

(c) Assume that only acoustic phonons near $q = 0$ contribute to heat transfer, such that their velocities determine $\langle v^2 \rangle$ in the expression for thermal conductivity. Determine the phonon mean-free time as well as phonon mean-free path at 20 K, 100 K, and 300 K.

8.4. Which material feels colder? (3 P)

Compare thermal effusivities of quartz and gypsum ($CaSO_4 \cdot 2H_2O$) at room temperature using their thermal conductivities of $8.0\,\mathrm{W/m\,K}$ and $0.2\,\mathrm{W/m\,K}$, respectively. This will give you a rough comparison between the feel of glass and plaster at home. Assume that heat capacity reaches the Dulong-Petit limit.