Problem sheet 9: Drude and Drude-Sommerfeld models **13.12.2023**

9.1. Think like Drude, calculate like Sommerfeld (6 P)

(a) Use experimental room-temperature resistivities for Au ($\rho = 2.1\,\mu\Omega\,\text{cm}$) and Ba ($\rho = 60\,\mu\Omega\,\text{cm}$) to determine mean-free time of the Drude model.

(b) Calculate Fermi energy (in eV) and Fermi velocity for both metals (Drude-Sommerfeld model).

(c) Take Fermi velocity as an estimate for the electron velocity. Calculate mean-free path for both metals. How does it compare to the interatomic distance?

9.2. Double names: Wiedemann-Franz from Drude-Sommerfeld perspective (3 P)

Wiedemann-Franz law postulates that the ratio of thermal conductivity (κ) and electrical conductivity (σ) is proportional to temperature, $\kappa/\sigma = LT$.

(a) Verify this relation using σ from the Drude model (classical electron gas) and heat capacity as well as electron velocity from the Drude-Sommerfeld model (Fermi gas). Take Fermi velocity as the average electron velocity.

(b) What is the value of the Lorenz number L?

9.3. Electronic specific heat and thermal conductivity (6 P)

Consider experimental data for the molar heat capacity of Al:

T (K)	c_p (mJ/mol K)	T (K)	c_p (mJ/mol K)
1.5	2.35	5	10.64
2.0	3.18	6	14.70
2.5	4.11	8	25.86
3.0	5.01	10	41.81
4.0	7.53	12	63.42

(a) Assume $c_p = \gamma T + \beta T^3$ at $T \to 0$. Determine the Sommerfeld coefficient γ.

Hint: the most accurate result can be obtained by plotting the data in Excel or another program and making the linear fit

(b) What is the density of states at the Fermi level? (Give the value in eV^{-1}). Compare the value of $g(\varepsilon_F)$ extracted from the experimental γ to the value expected for the Fermi gas with the electron concentration of Al.

(c) Determine electronic contribution to the thermal conductivity of Al at 295 K using the mean-free time $\tau = 7.5 \times 10^{-15}$ s. Take Fermi velocity as the mean electron velocity.

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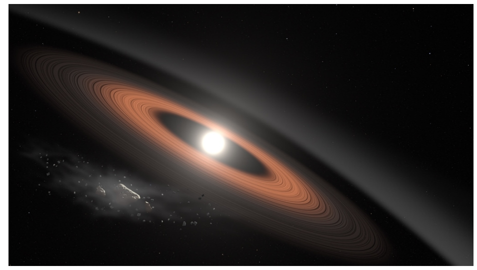
9.4. Star problem (5 P)



Astrophysicists predict that in 5 billion years the Sun will fuse all of its hydrogen and become a white dwarf with the radius of $R = 10\,000\,\text{km}$ and half of the current mass, $M = \frac{1}{2}M_\odot$ ($M_\odot = 1.99 \times 10^{30}\,\text{kg}$).

(a) Assume that the white dwarf consists of helium. Determine the effective radius of an atom in this highly compressed state. Compare it to the standard radius of a free helium atom.

(b) High density leads to the full ionization of helium atoms. Determine the Fermi energy (in eV) and Fermi temperature of the corresponding electron gas.

(c) Determine the pressure of this Fermi gas at $T = 0$. Compare it to the pressure due to gravitational forces, GM^2/R^4 where G is the gravitational constant. You should find an order of magnitude difference between the two pressures. This difference is compensated by thermal pressure, as the white dwarfs are still very hot.

Credits:

- heat capacity data – Physica 4, 835–842 (1937)
- white dwarf – NASA (public domain)